

A new algorithm to optimize a can-order inventory policy for two companies in a horizontal partnership

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Agenda

- Horizontal cooperation: what, why, how?
- Examples of horizontal cooperations
- Definitions & assumptions
- Problem setting example
- Costs & performance measures
- Methodology
- Numerical example
- Future research

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Horizontal Cooperation

- **What** = cooperation where companies bundle their orders/join shipments
- **Why** = to reduce transport costs, CO_2 emissions, and congestion
- **How** = by using the available space in truck hauls of one company to ship items of another company
- Vertical cooperation = cooperation with companies at different level of the supply chain (e.g., supplier & buyers)
- Horizontal cooperation = cooperation with companies at the same level of the supply chain

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Examples of Horizontal Cooperation

Tupperware

P&G

Examples of Horizontal Cooperation

Tupperware

P&G

Nestlé

pepsi

Examples of Horizontal Cooperation

Tupperware

P&G

Nestlé

pepsi

ucb

Baxter

Examples of Horizontal Cooperation

What do we observe?

1. Horizontal cooperations can be established even with competitors!
2. Horizontal cooperations often only have 2 partners.

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Definitions & Assumptions

- Assumptions:
  - Two companies
  - Both companies adopt a (S,c,s) can-order policy to synchronize their orders
  - No replenishment lead time
  - Unit Poisson demand (iid for both companies)

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- Assumptions:
  - Two companies
  - Both companies adopt a (S,c,s) can-order policy to synchronize their orders
  - No replenishment lead time
  - Unit Poisson demand (iid for both companies)
- Definitions:
  - I_i = the inventory level at company i
  - S_i = the order-up to level of company i
  - c_i = the can-order level of company i
  - s_i = the reorder-point of company i
  - λ_i = the Poisson arrival rate of customers at company i

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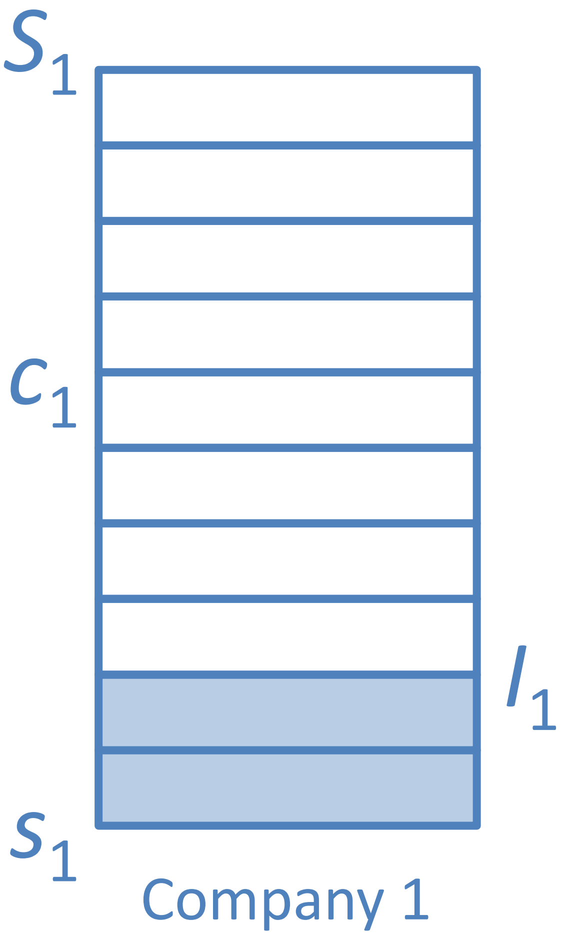
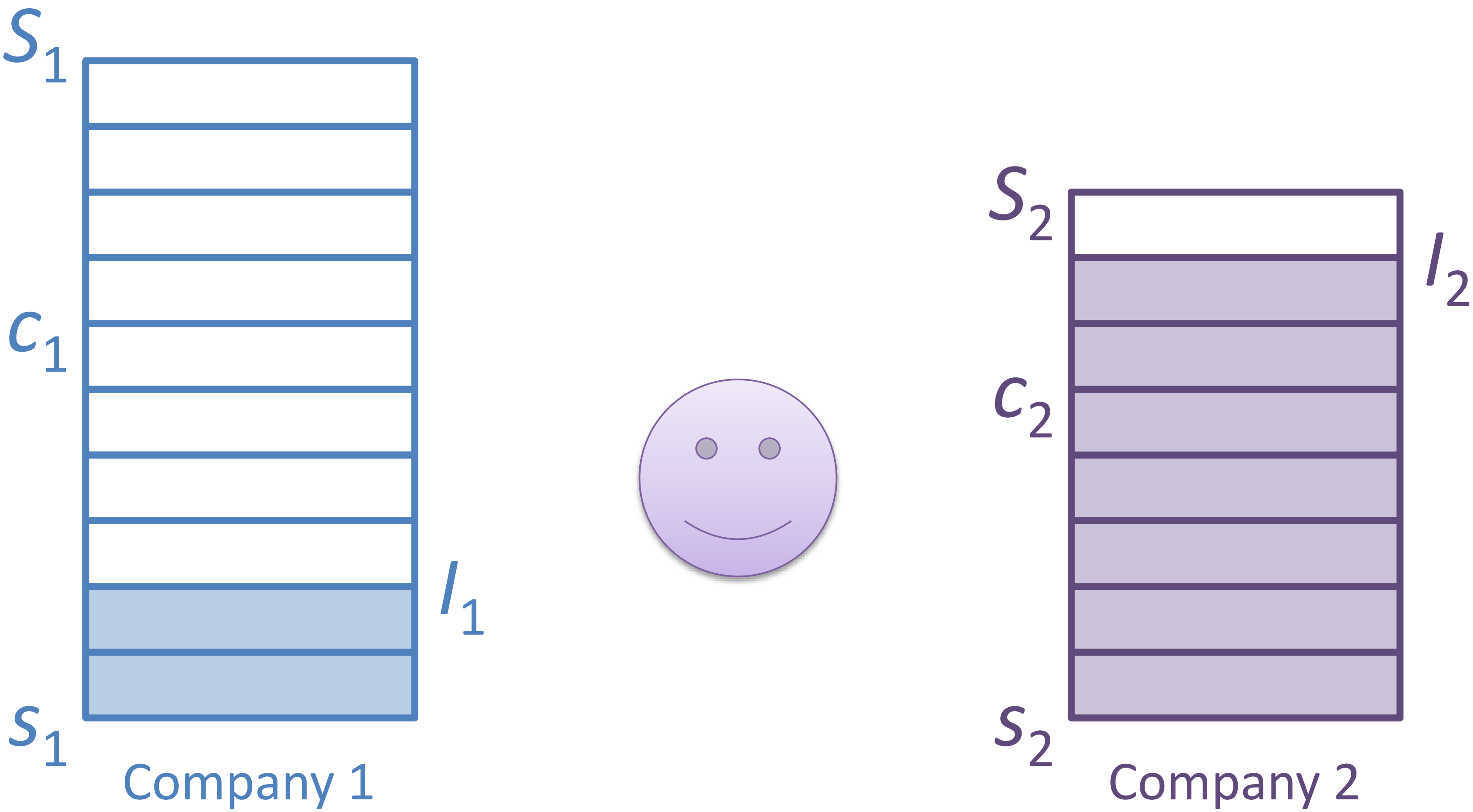


Problem Setting Example ($t = t_1$)



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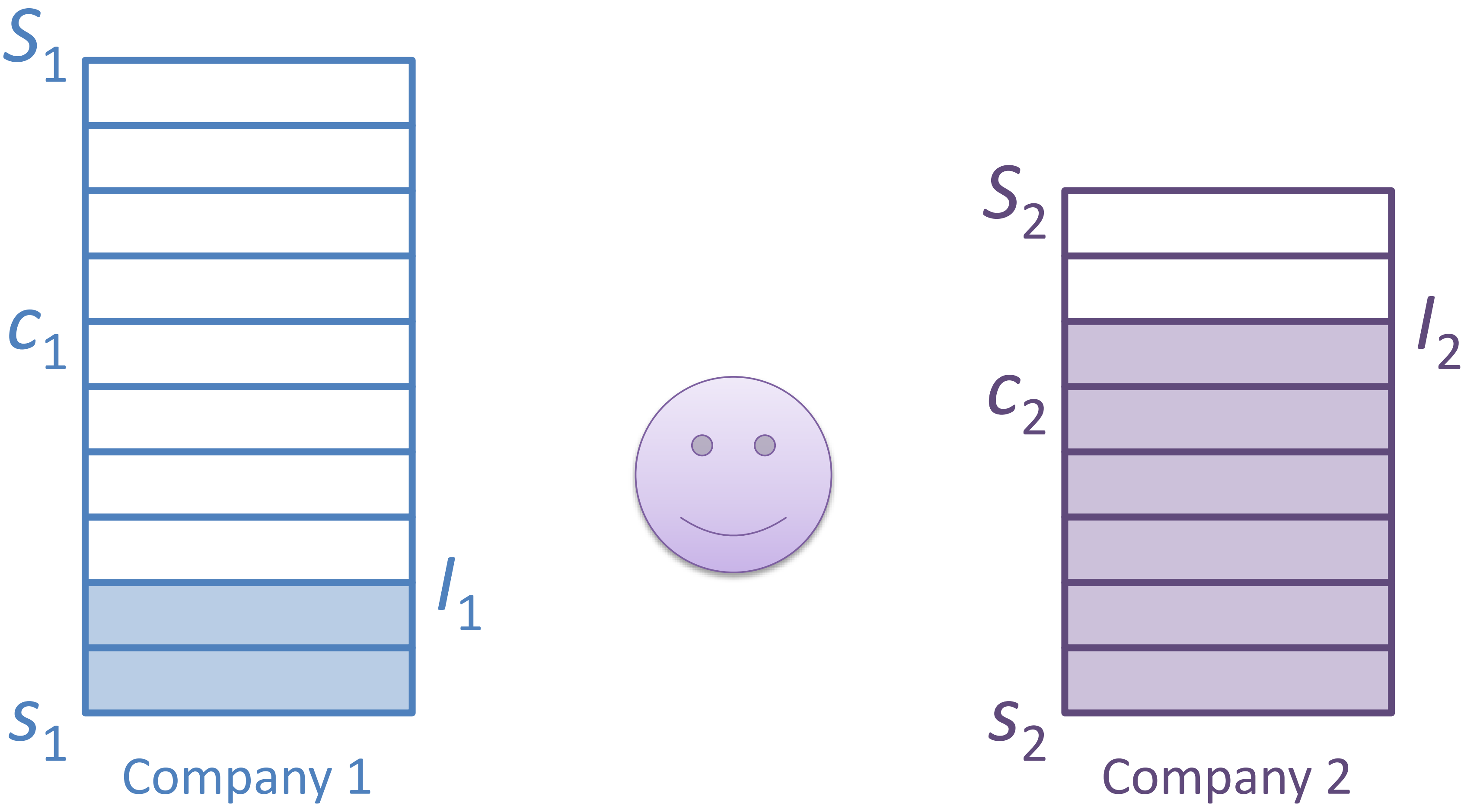
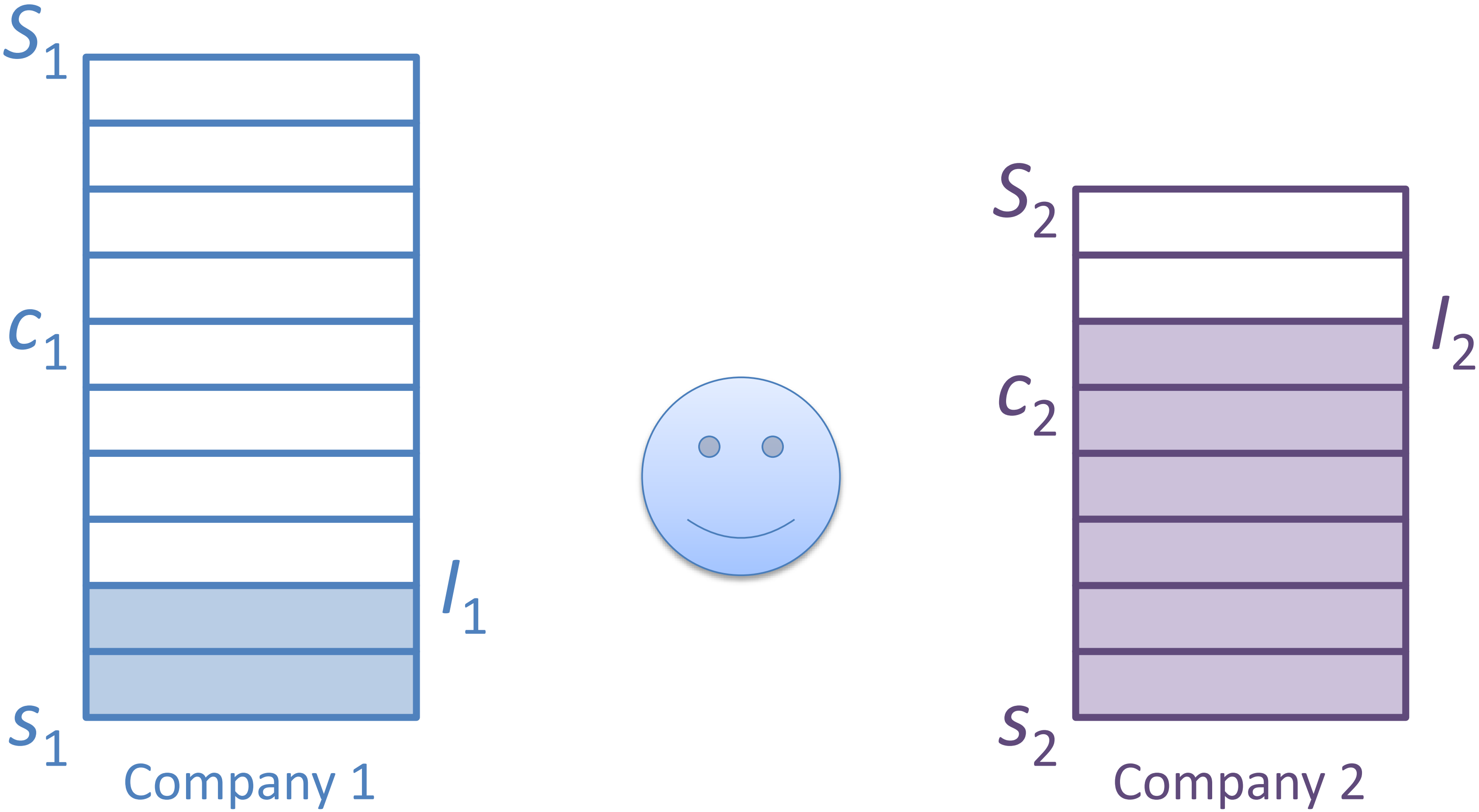


Problem Setting Example ($t = t_2$)



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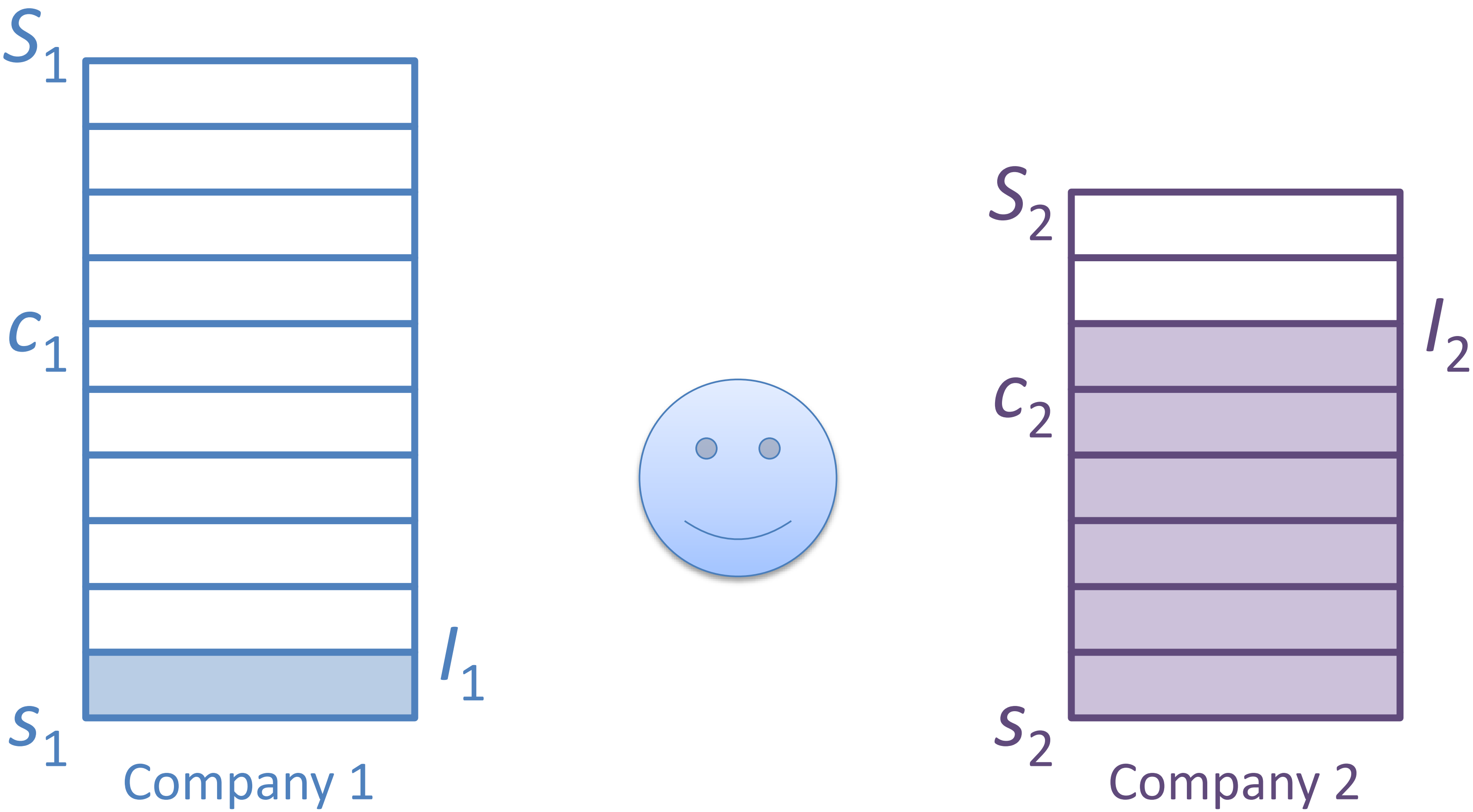

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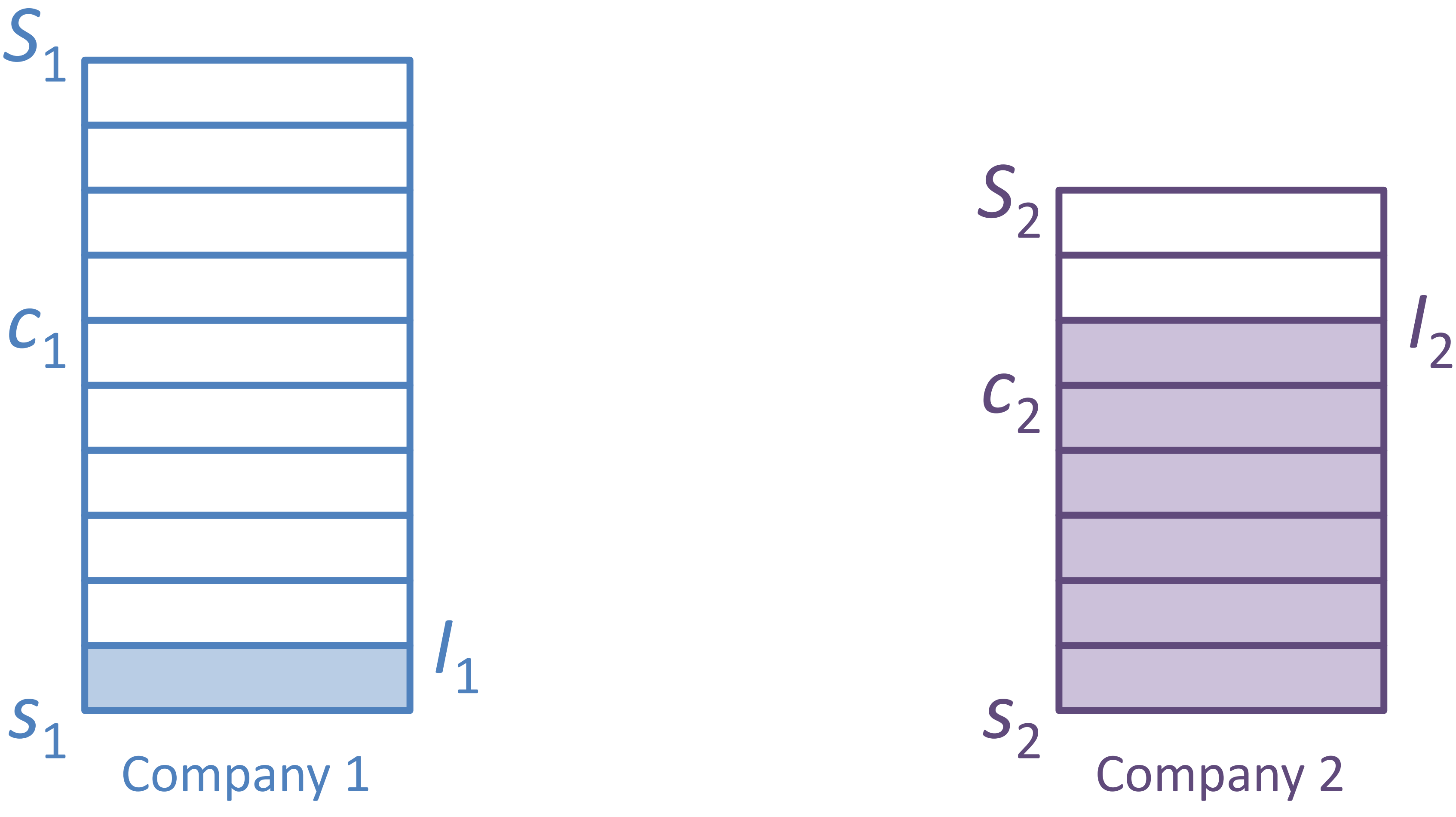

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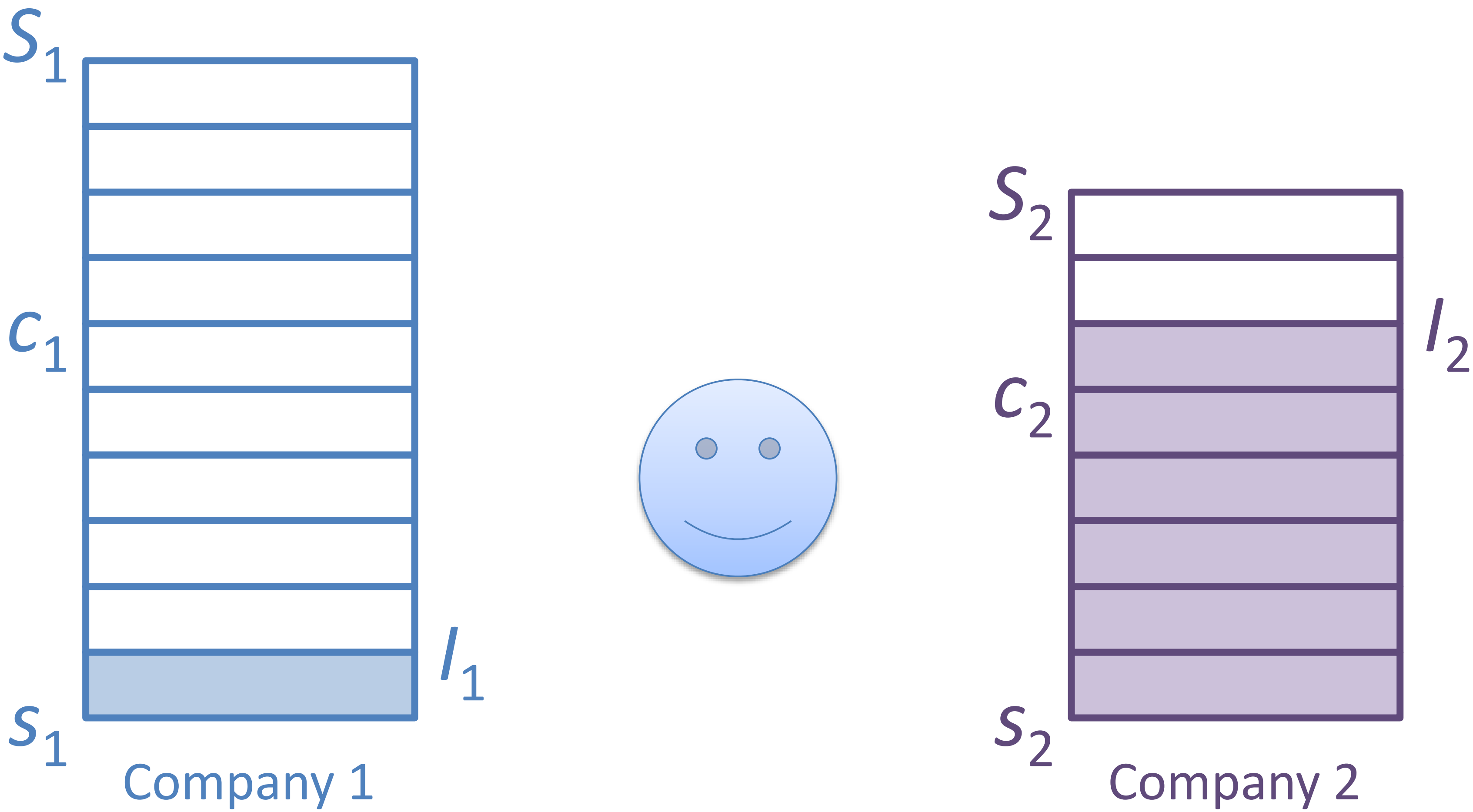

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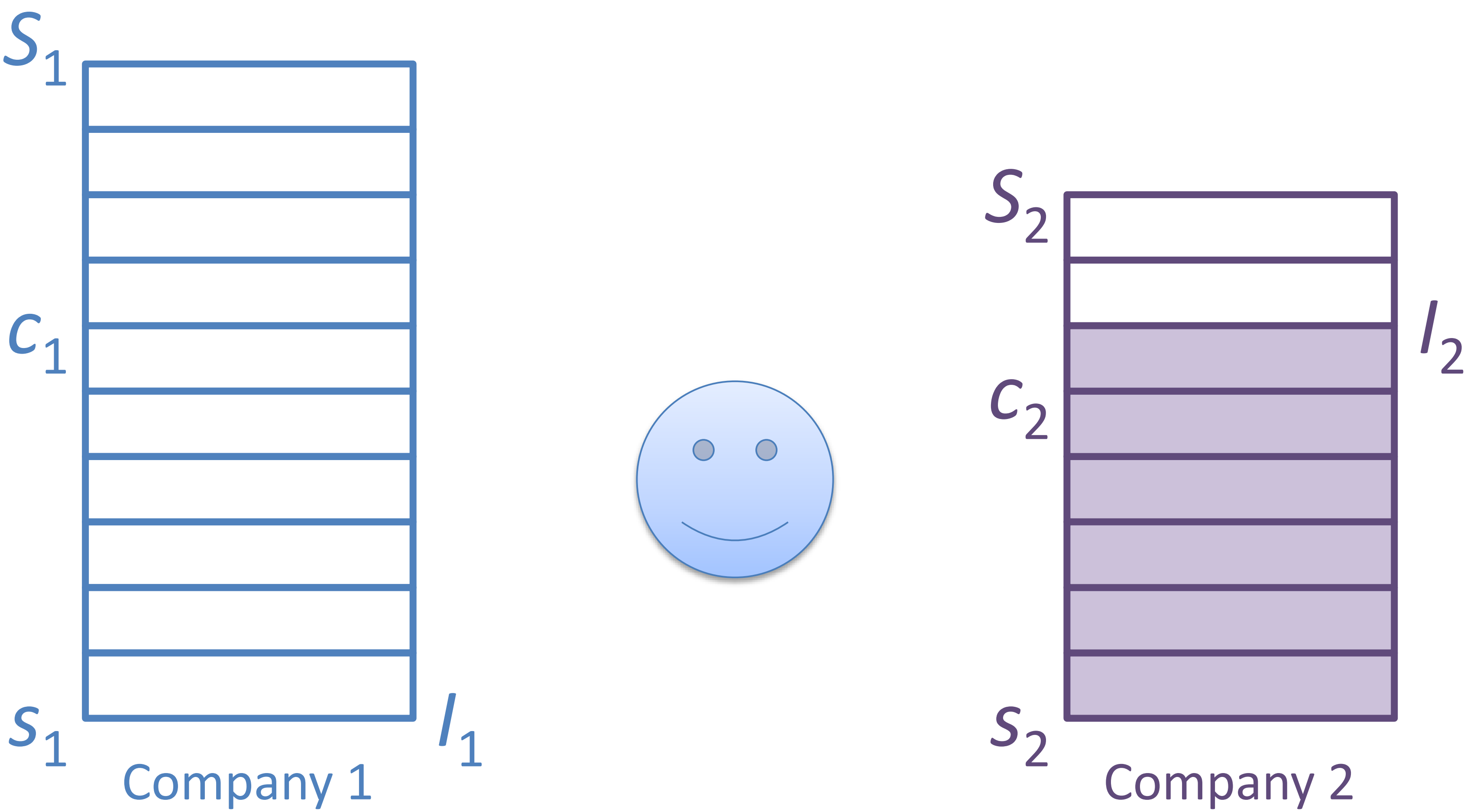

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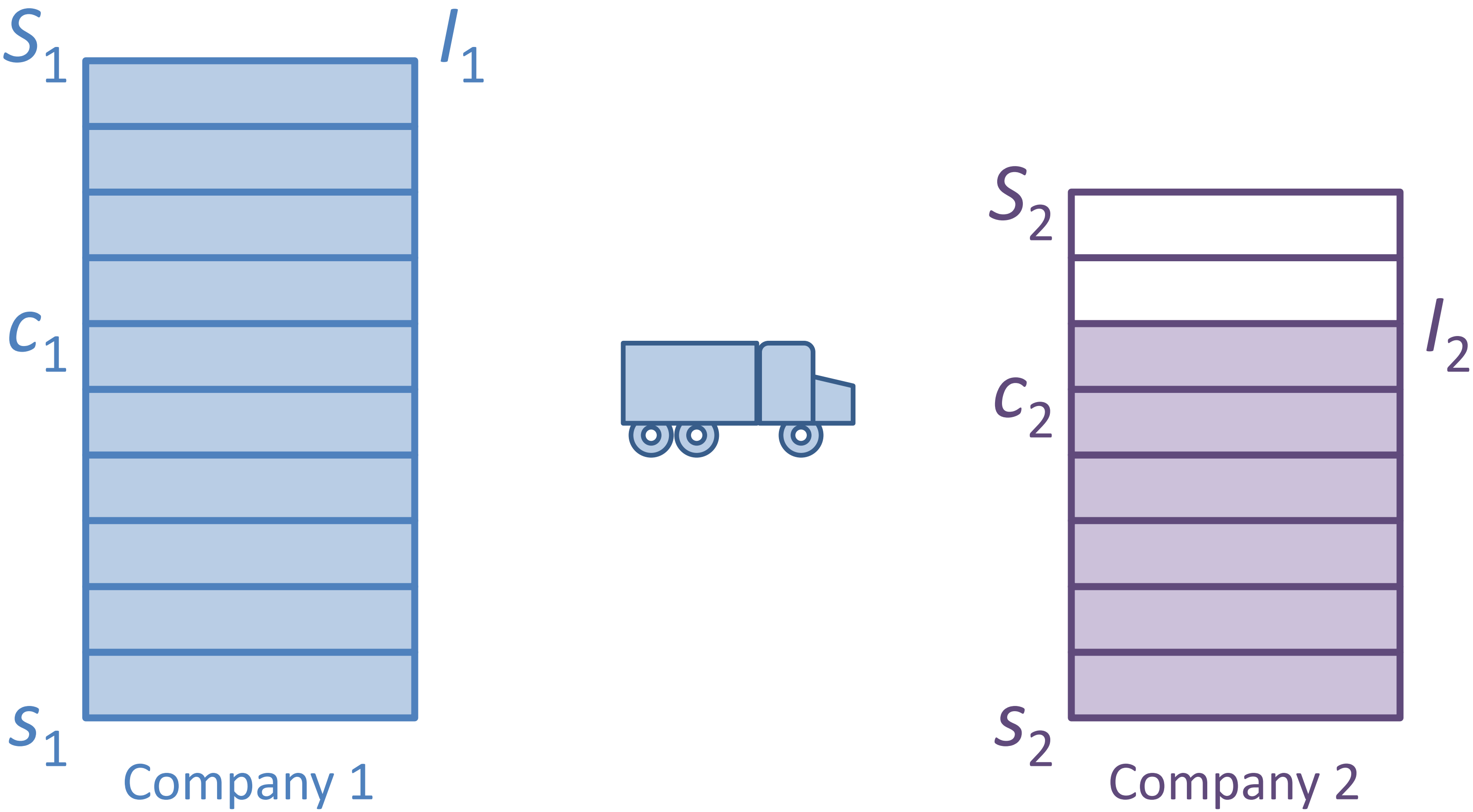

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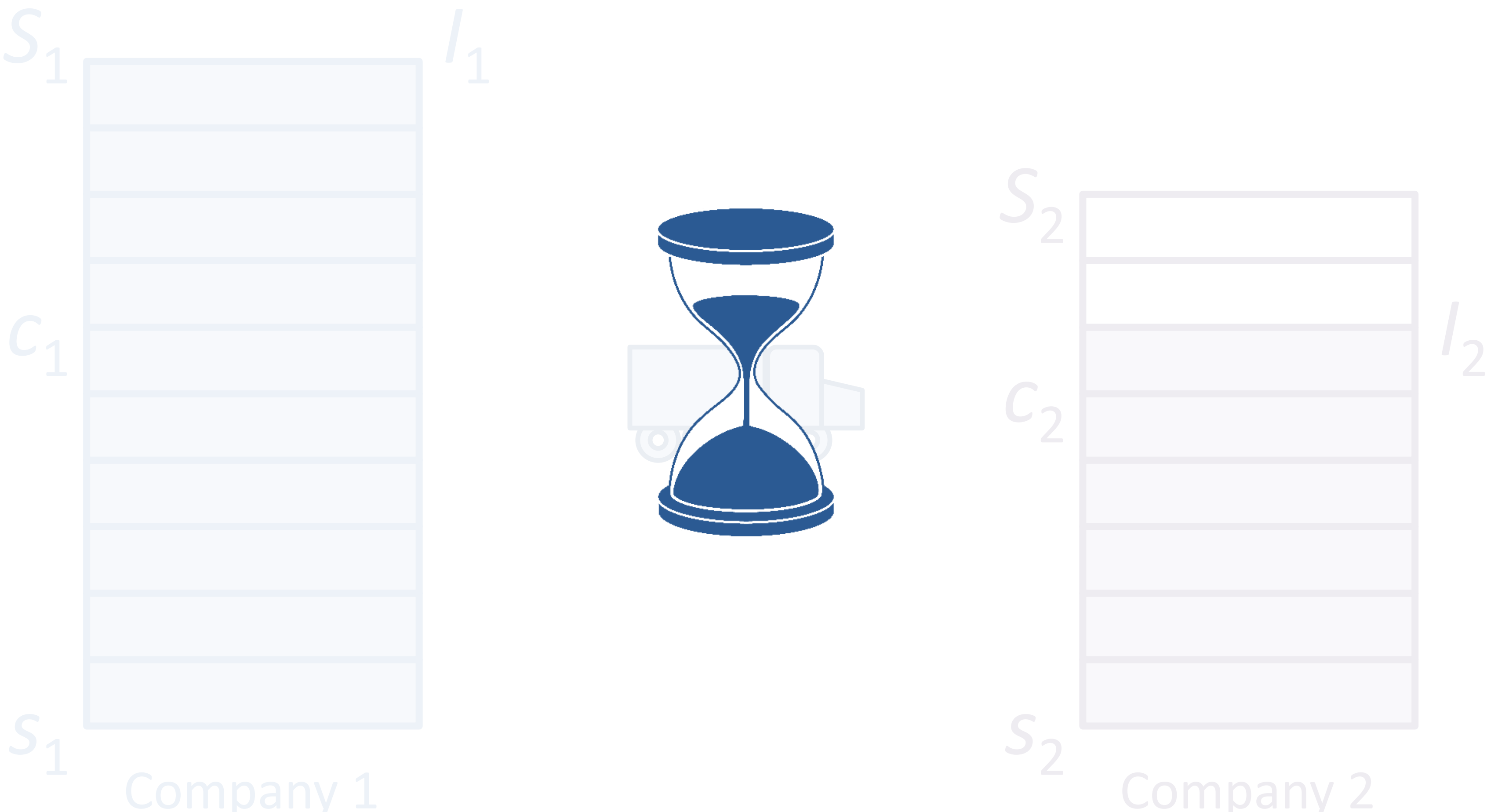
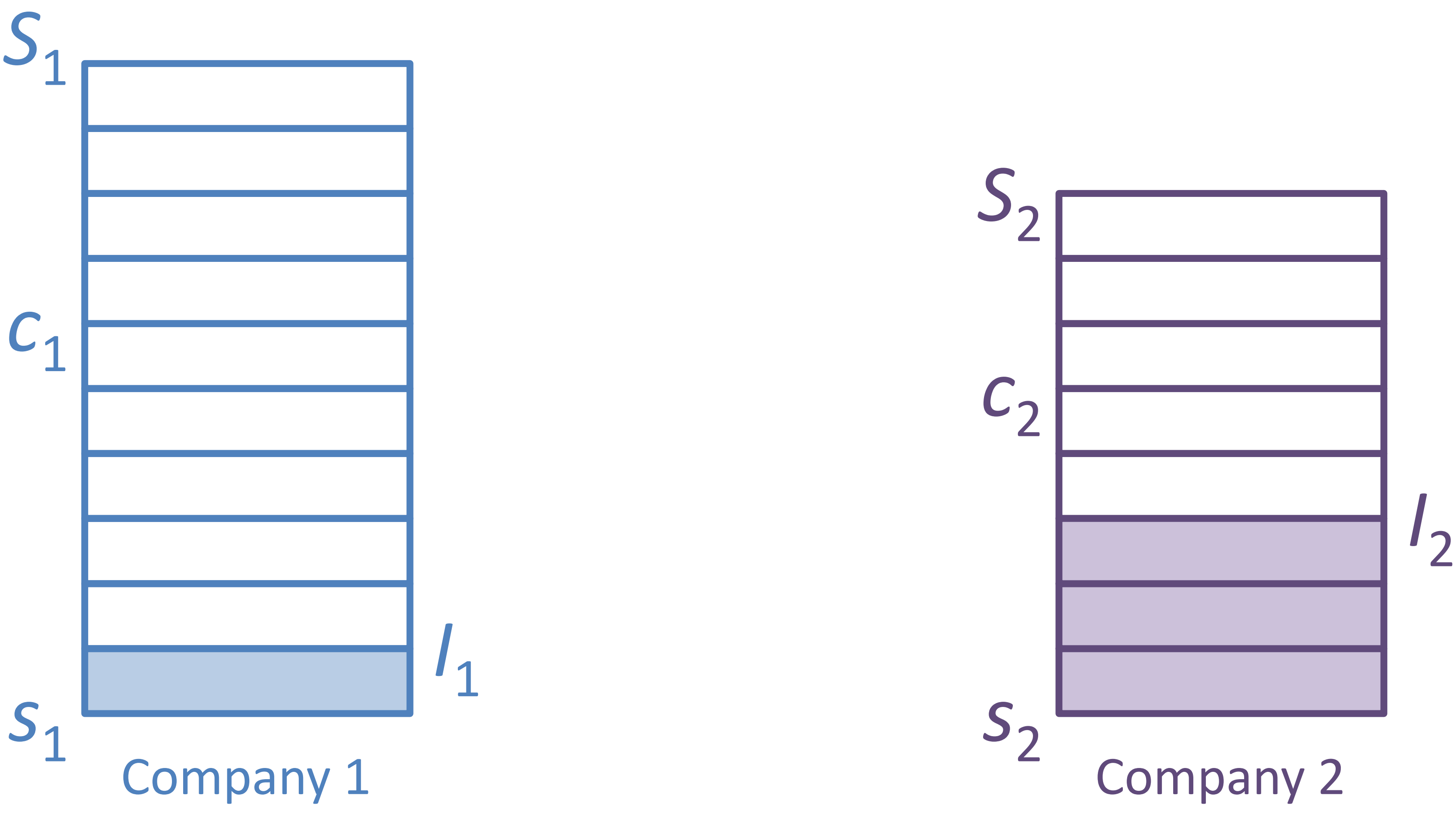


Problem Setting Example ($t = t_x$)



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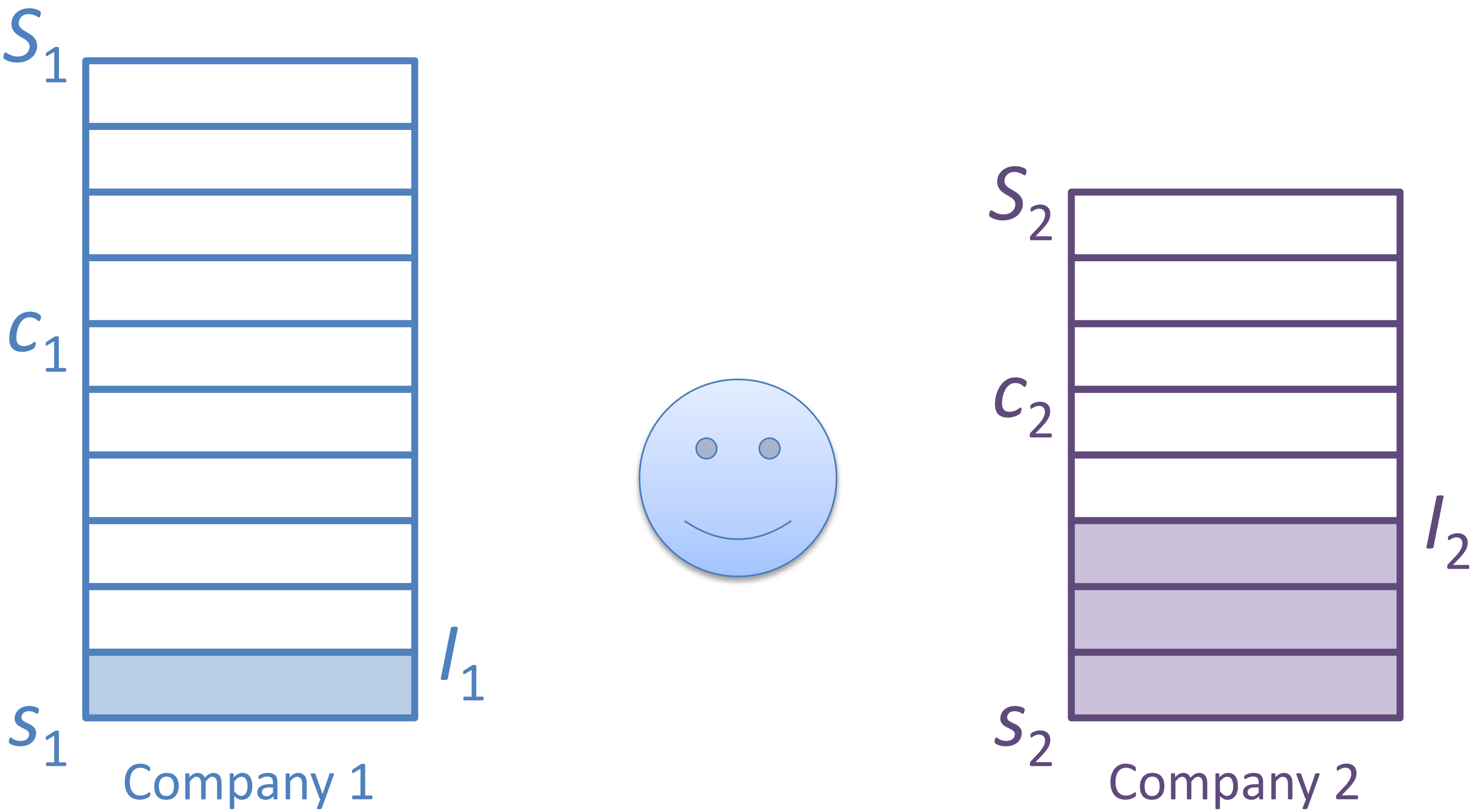

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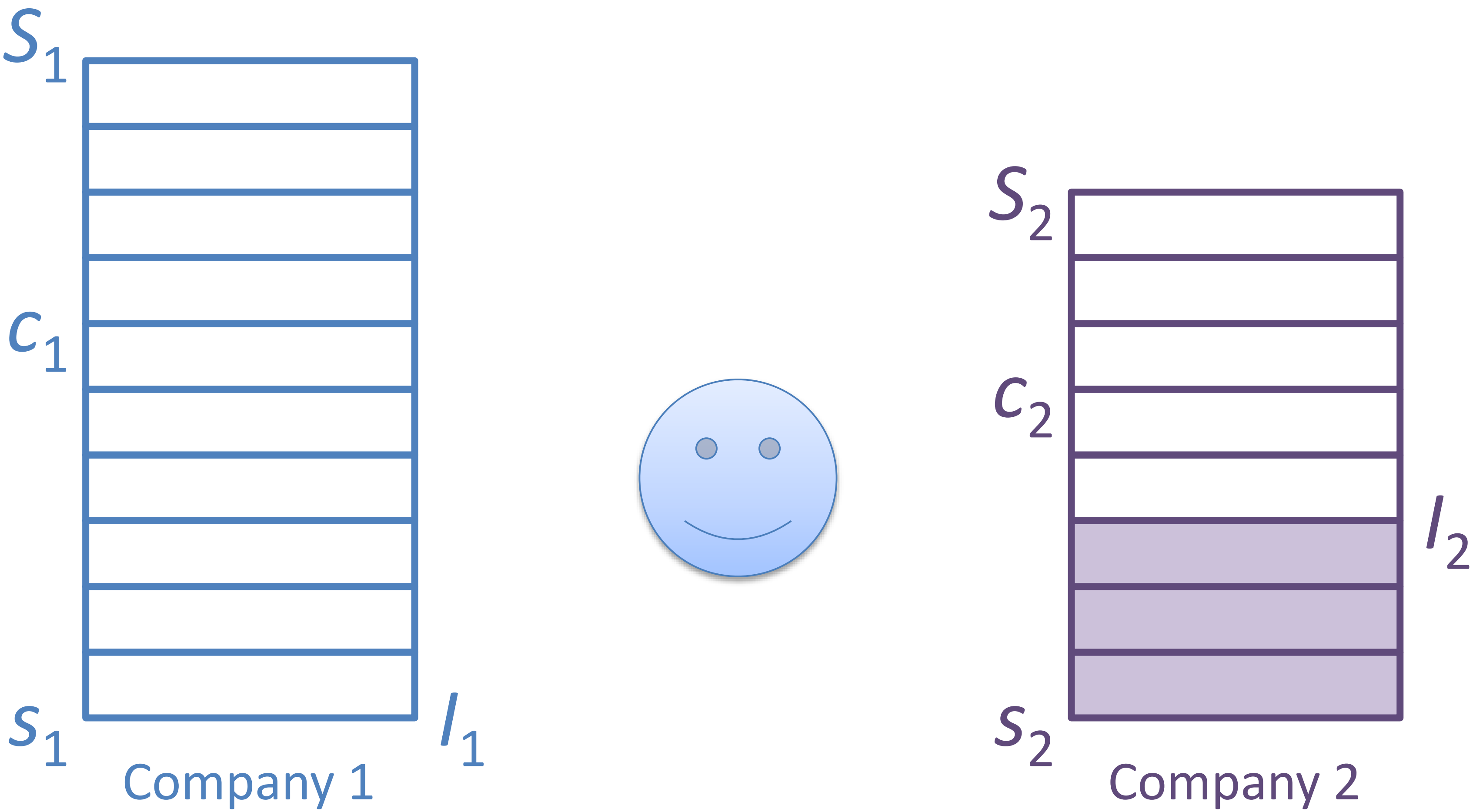

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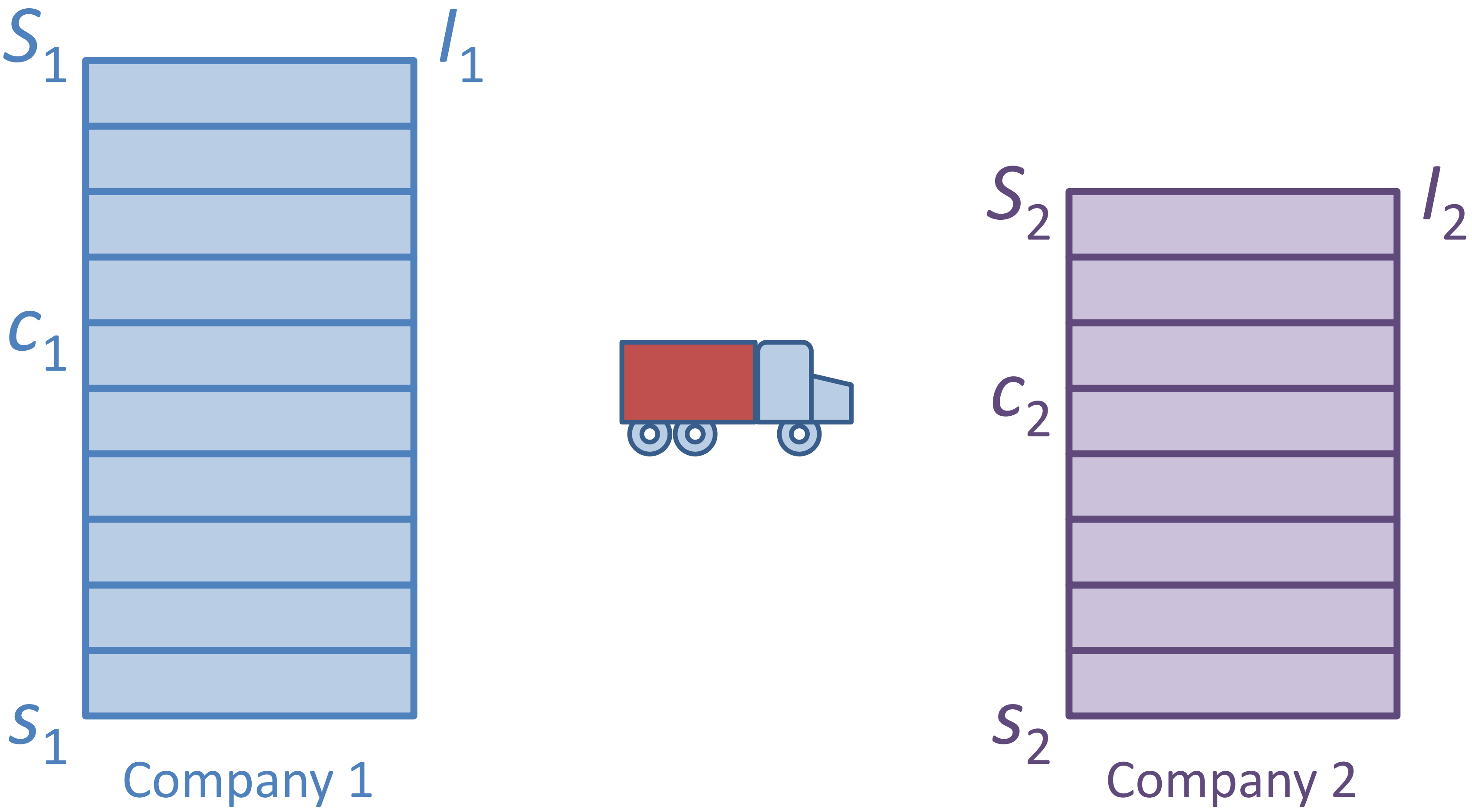

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$\Rightarrow$ The total cost for both company given their (S,c,s) policy

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- **How**:
  1. Evaluate the performance of a single (S,c,s) policy
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- **Available methodologies**:
  - Simulation
  - Markov chains
  - A new approach?

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  $\Rightarrow$ 500 times smaller!

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Assume we start from a “full” system

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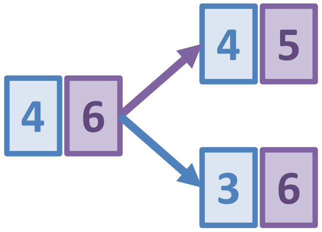

- There is a 10% probability that the next customer visits company 1
- There is a 90% probability that the next customer visits company 2

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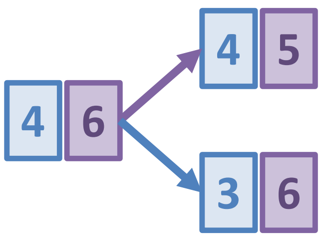
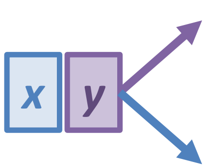


Law of competing exponentials



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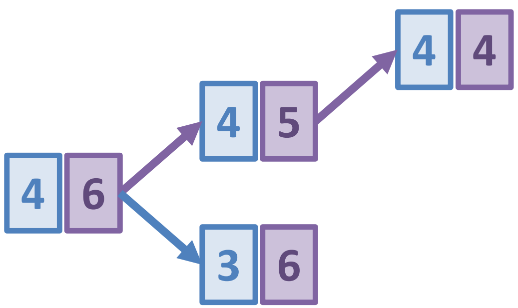

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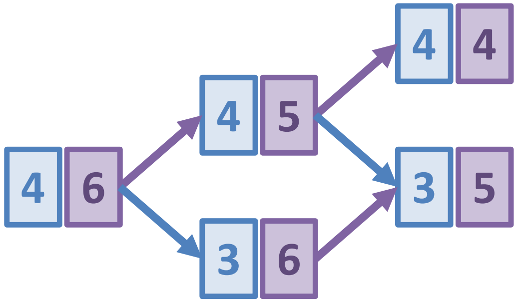

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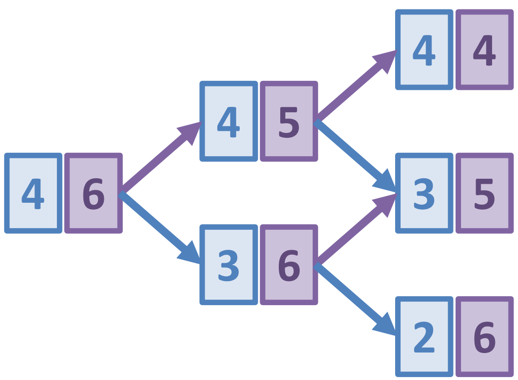

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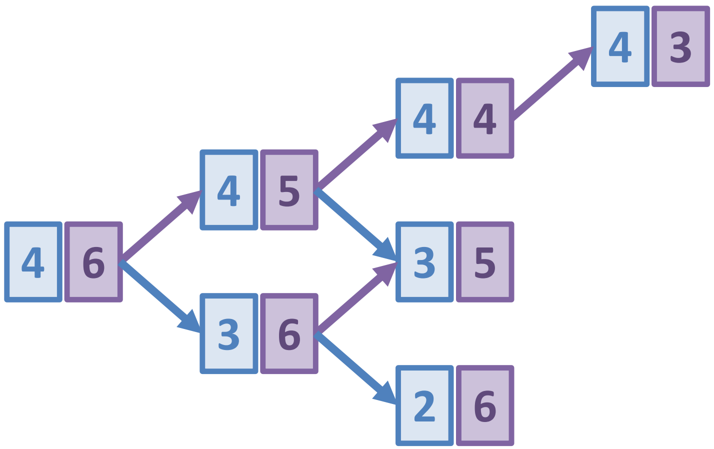

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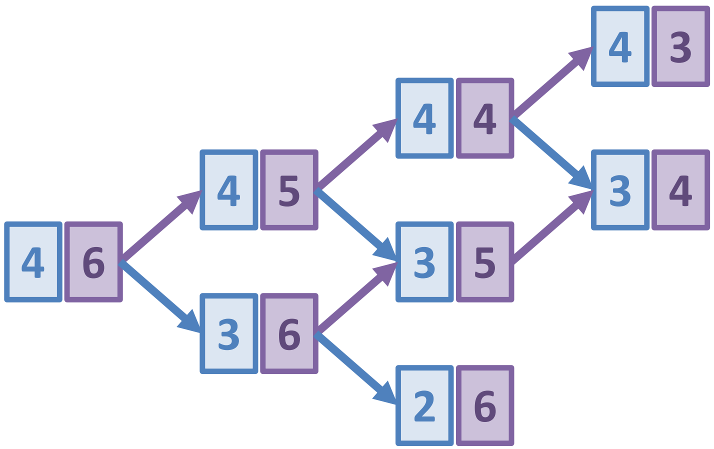

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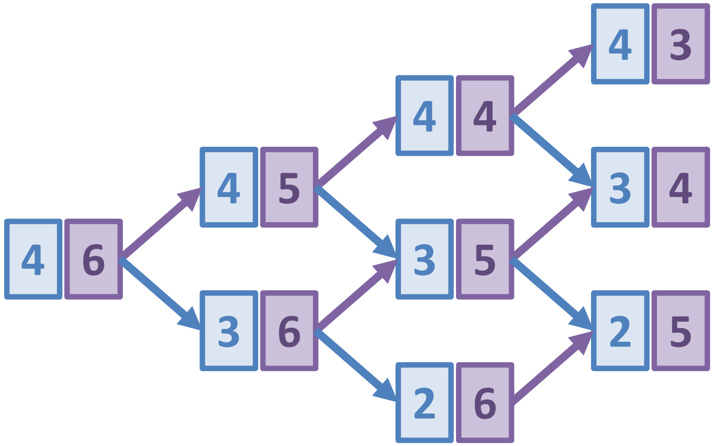

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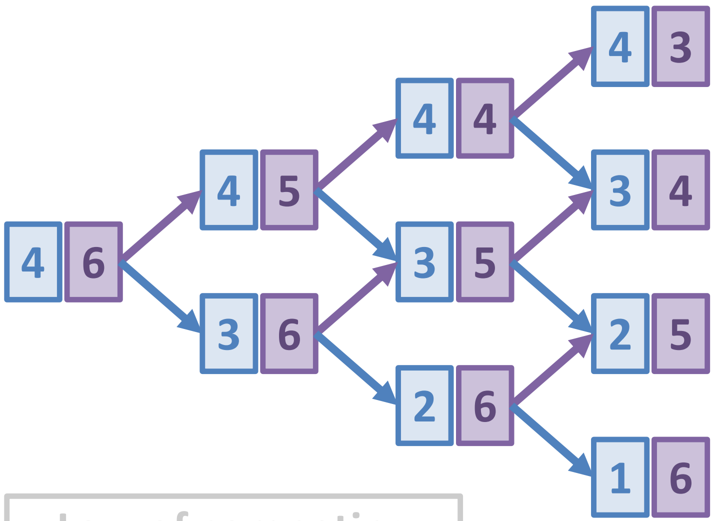

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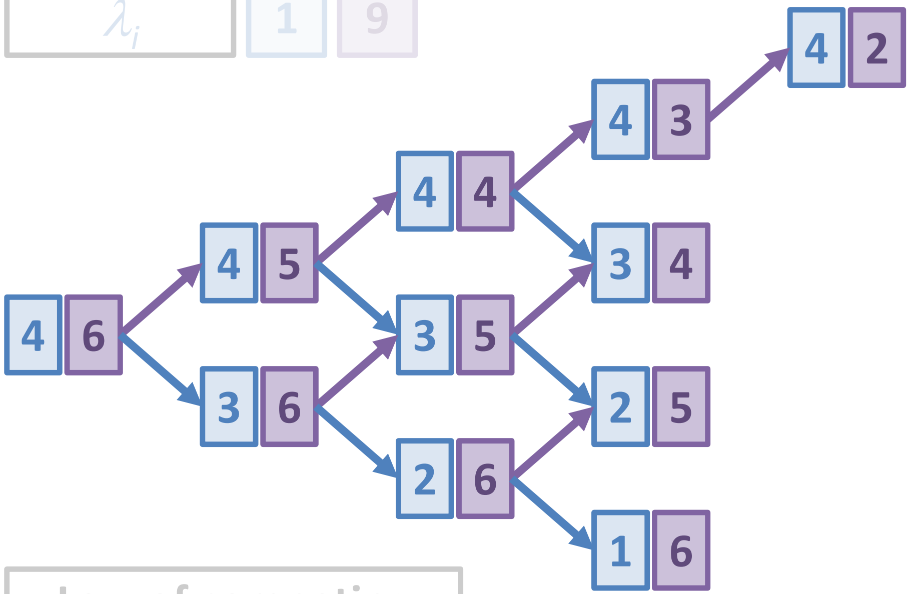

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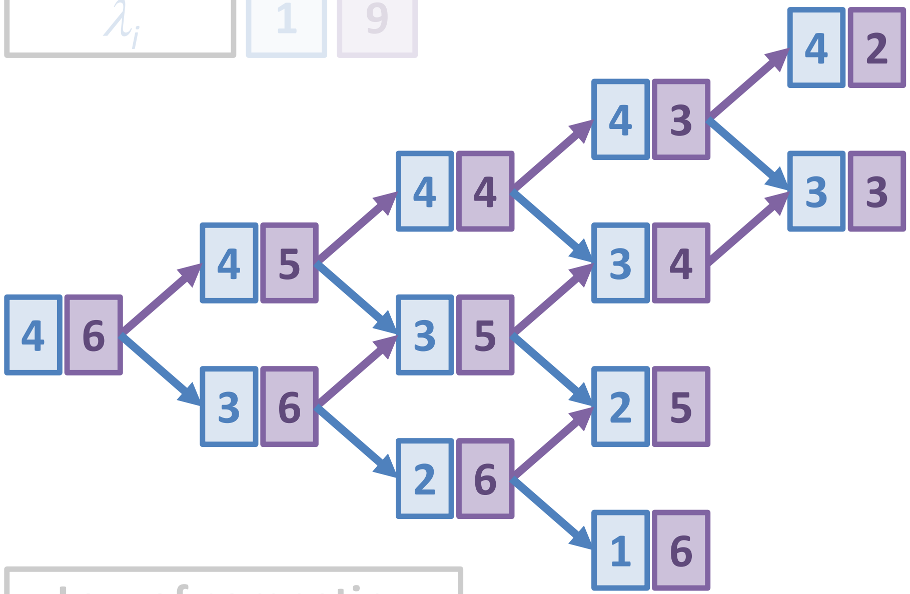

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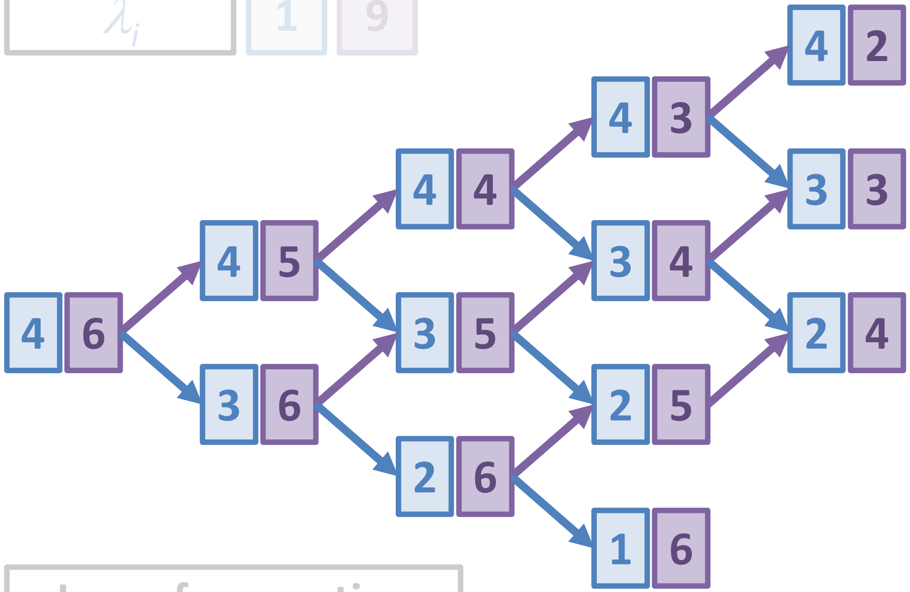

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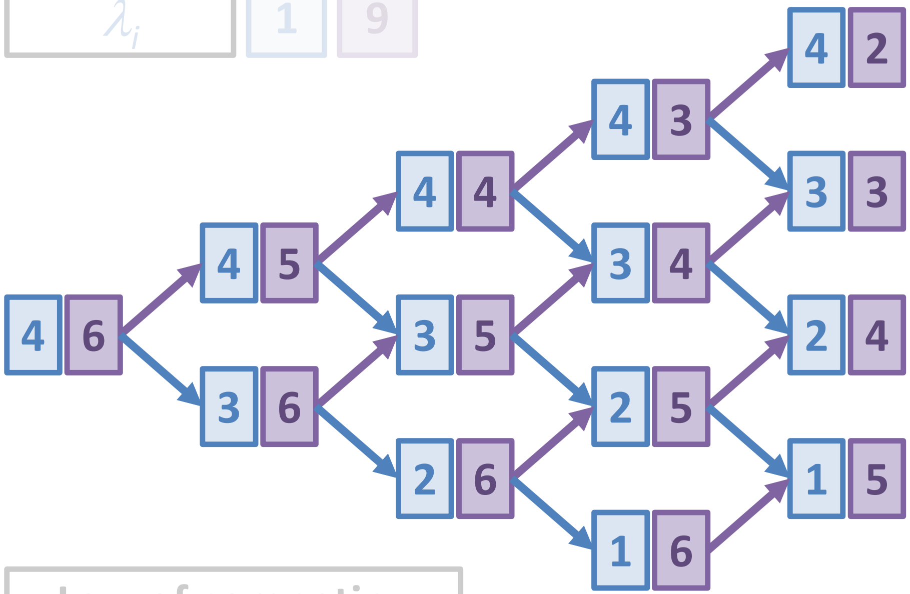

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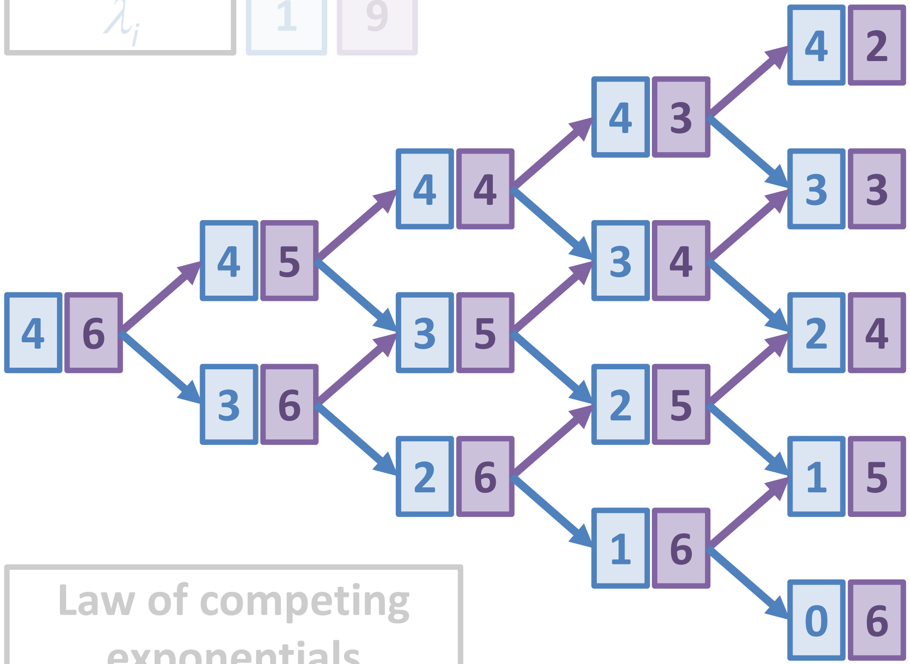

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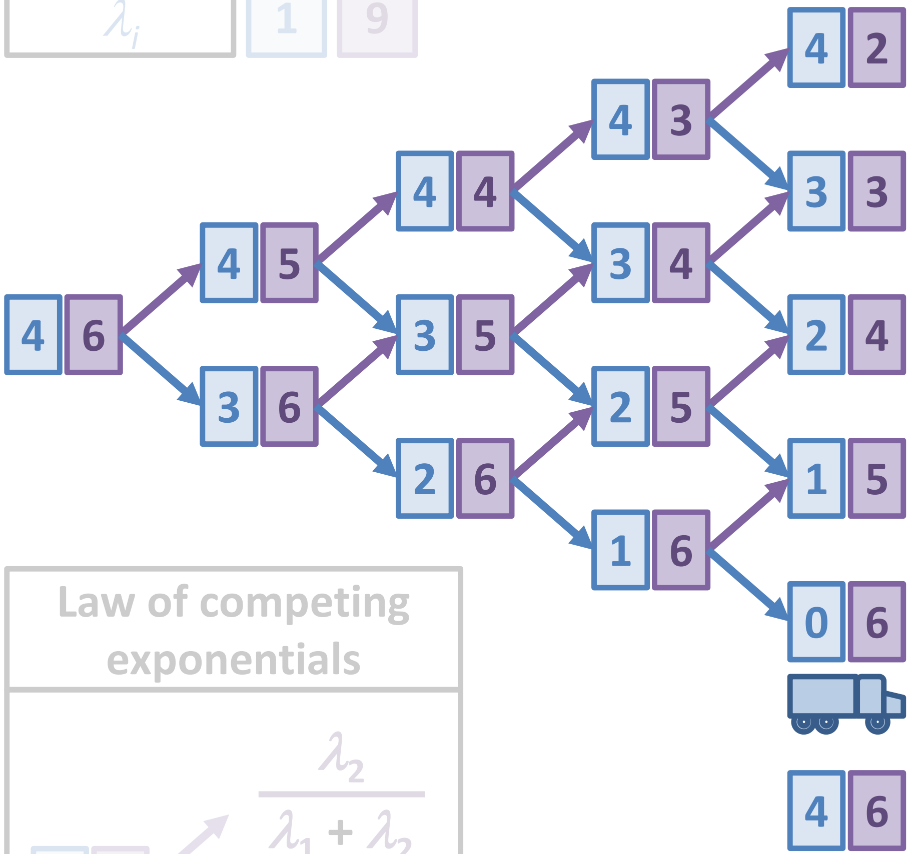

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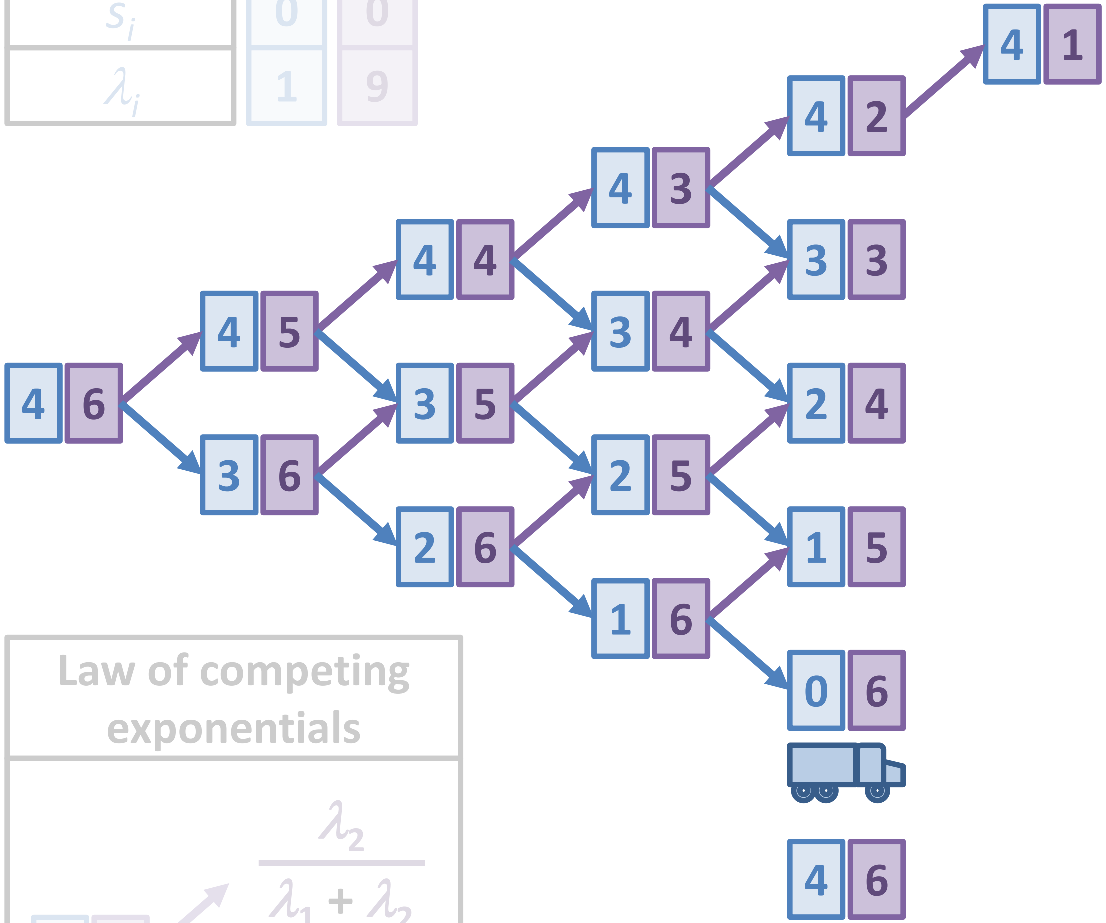

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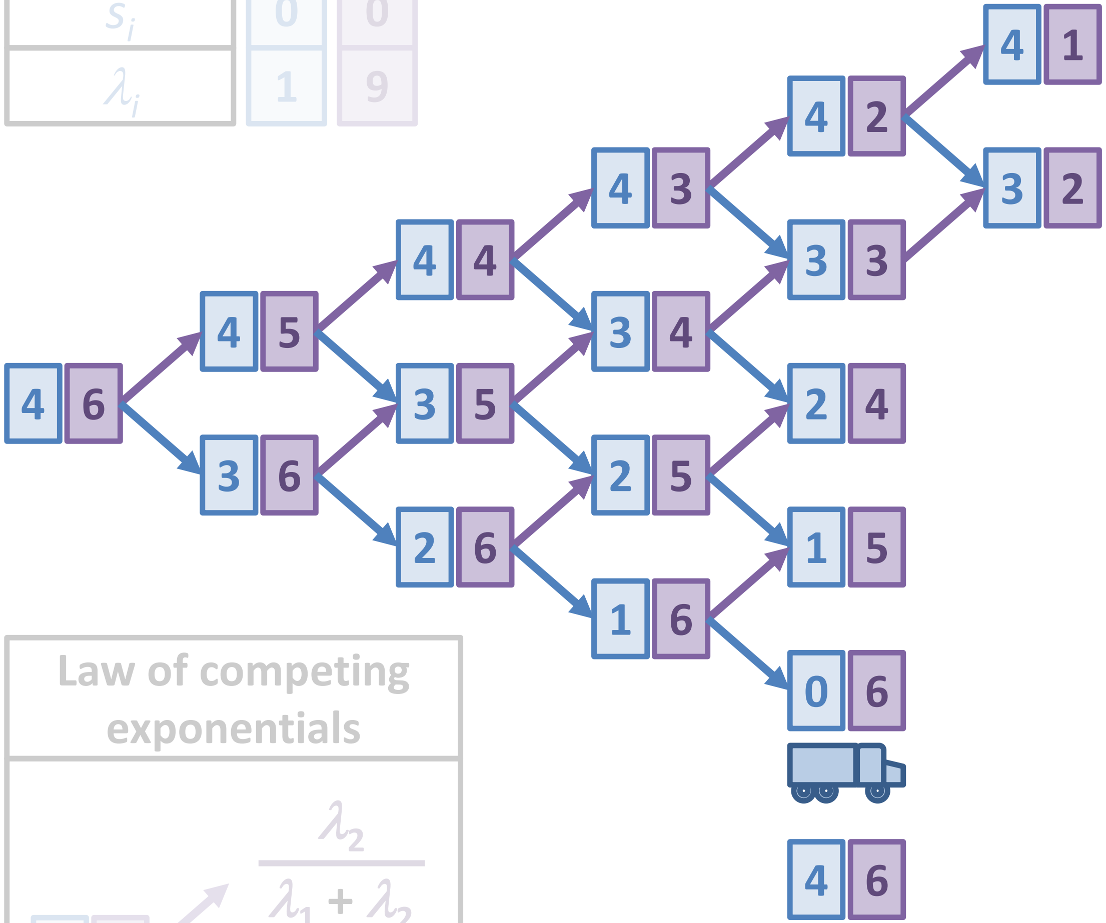

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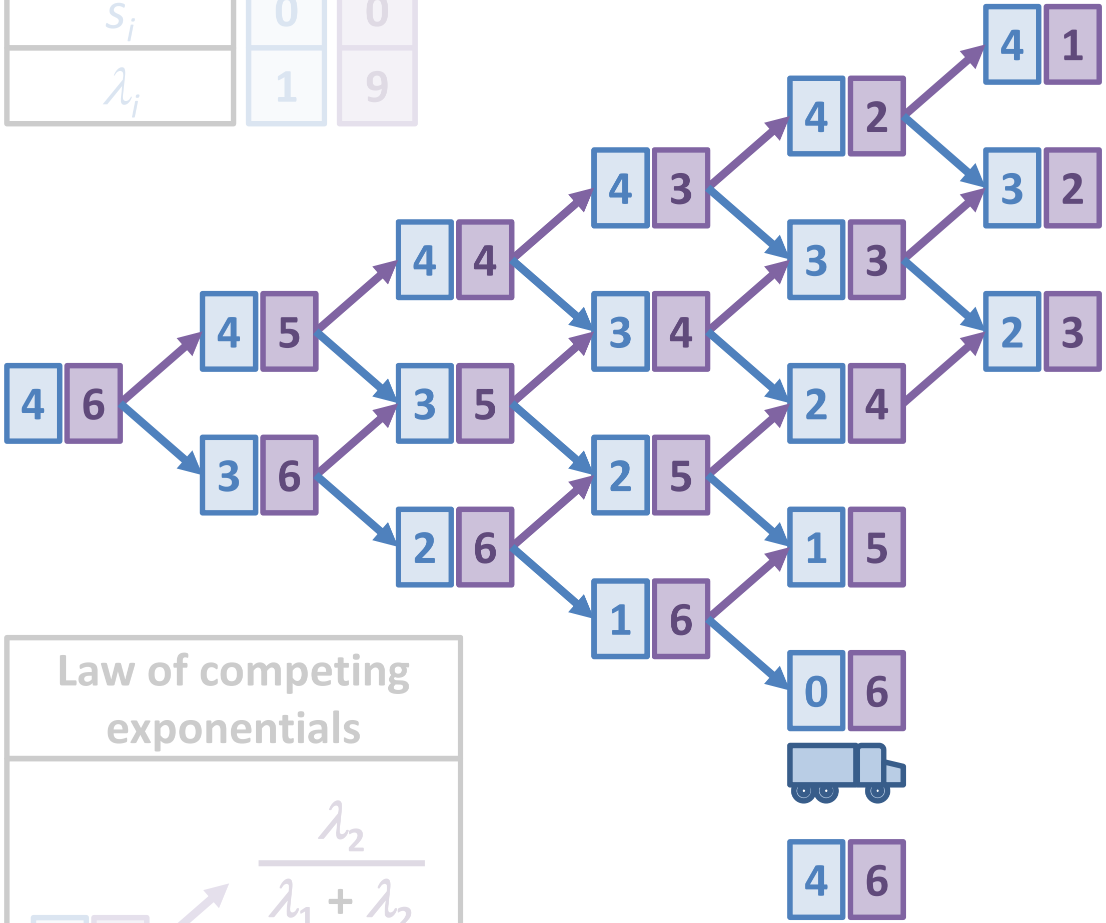

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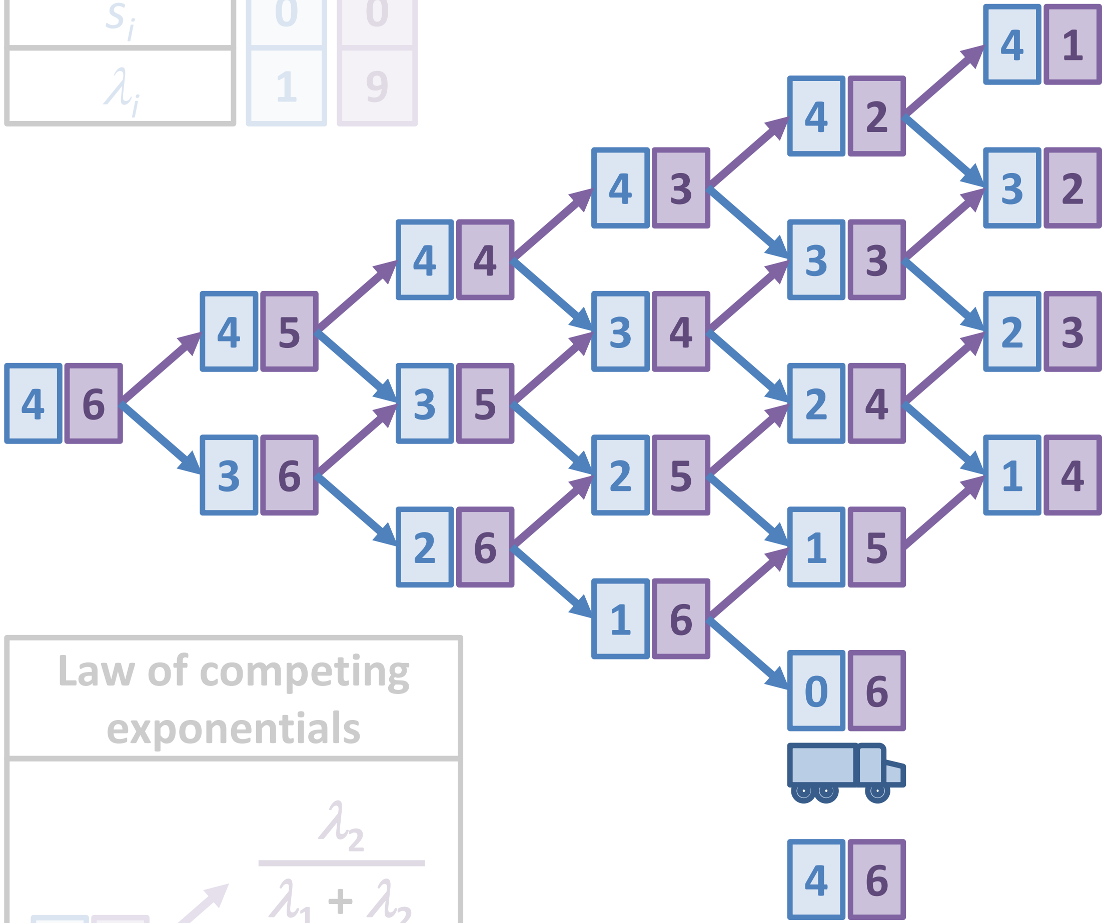

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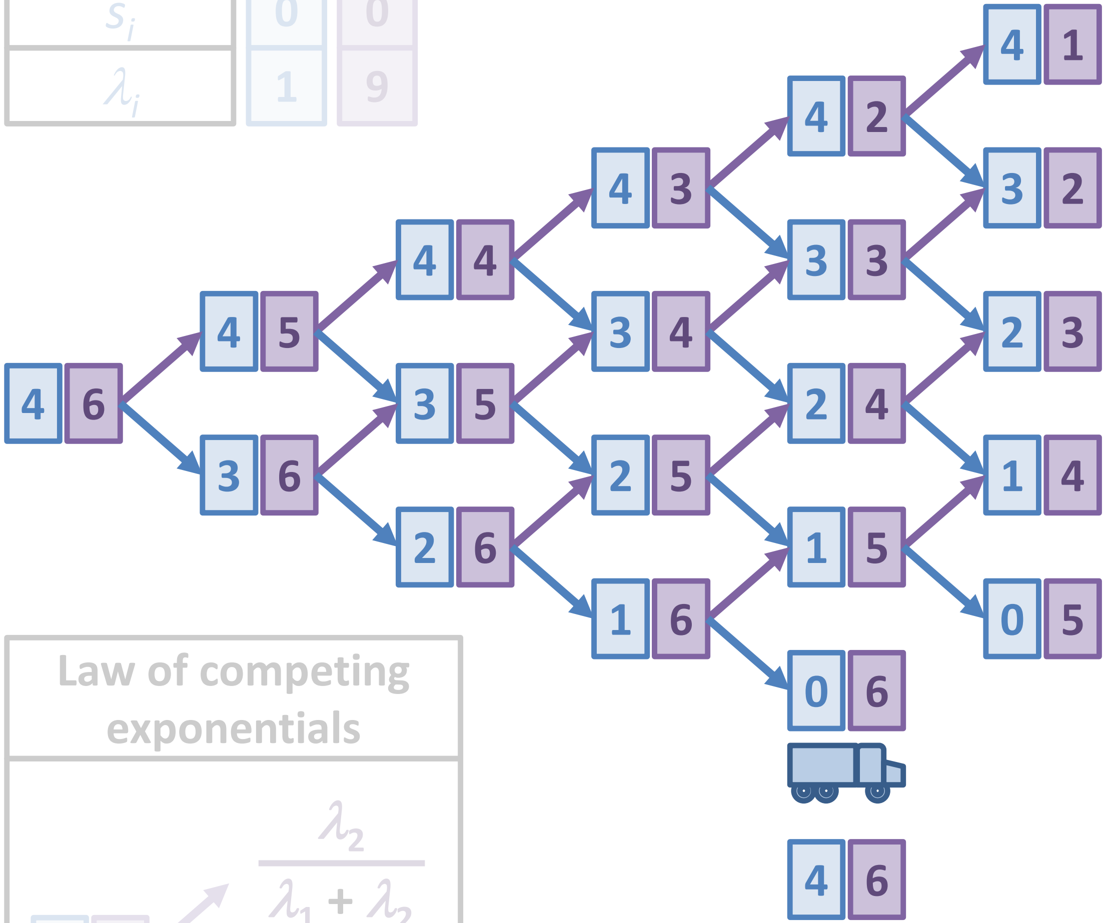

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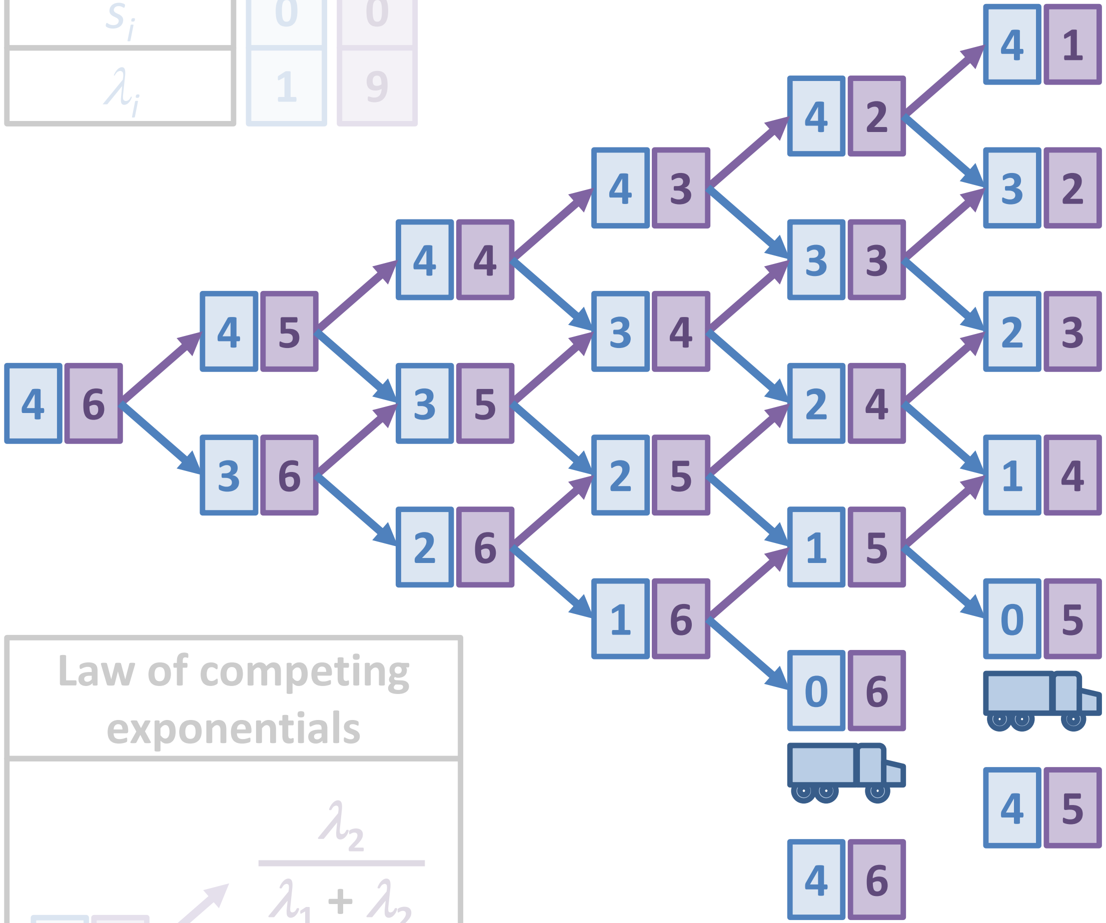
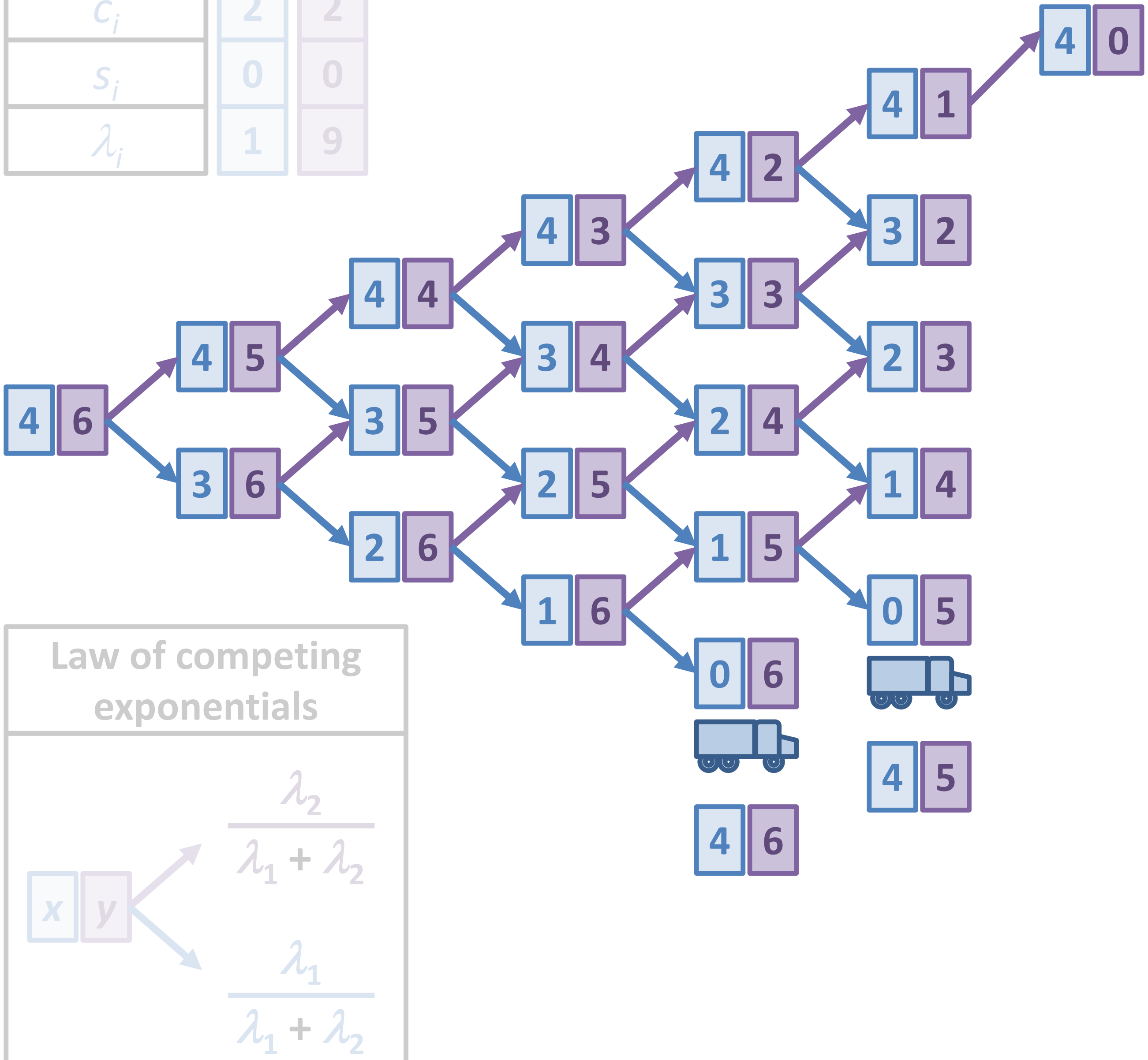
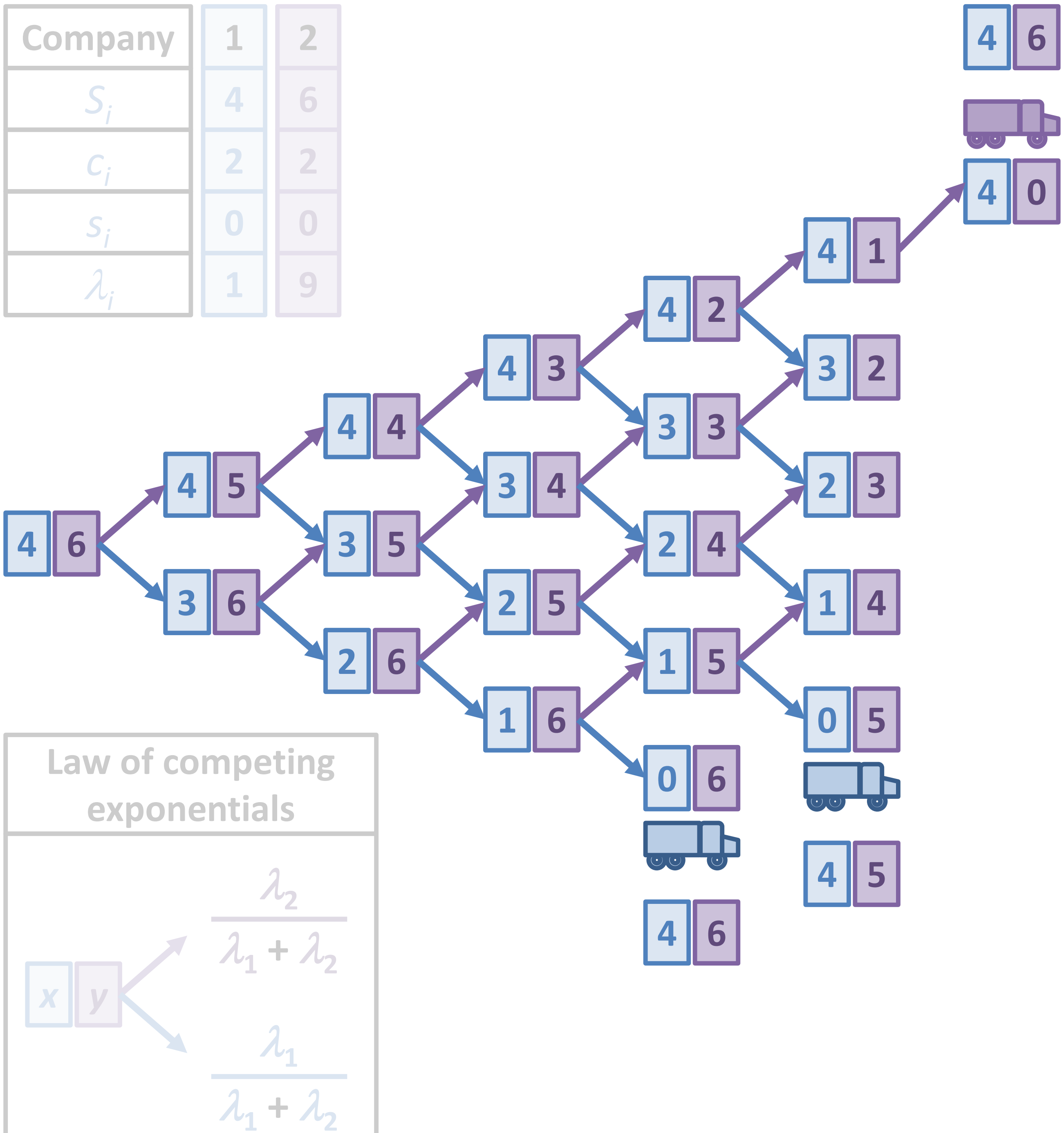


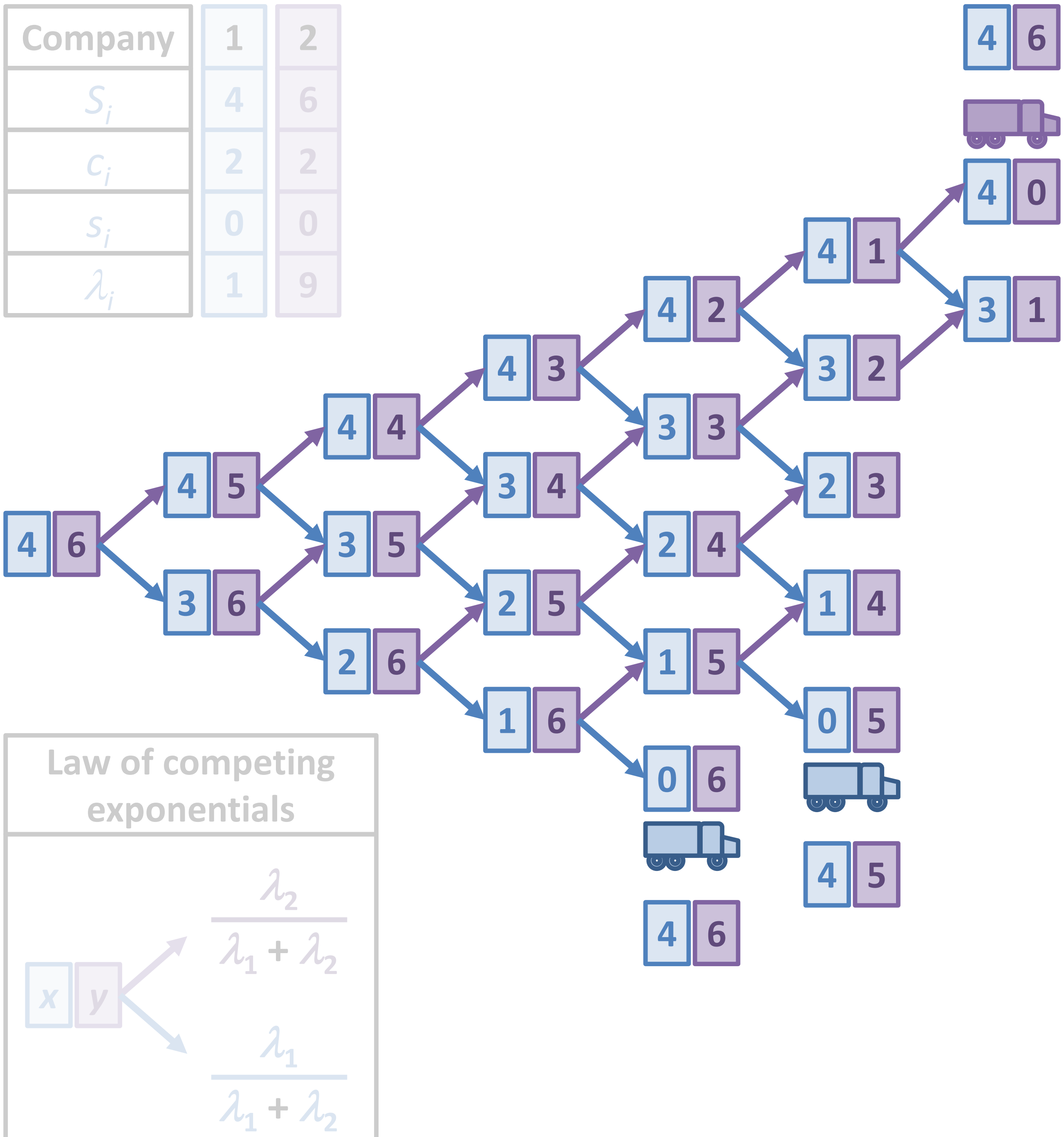
Law of competing exponentials

Company	1	2
S_i	4	6
c_i	2	2
s_i	0	0
λ_i	1	9



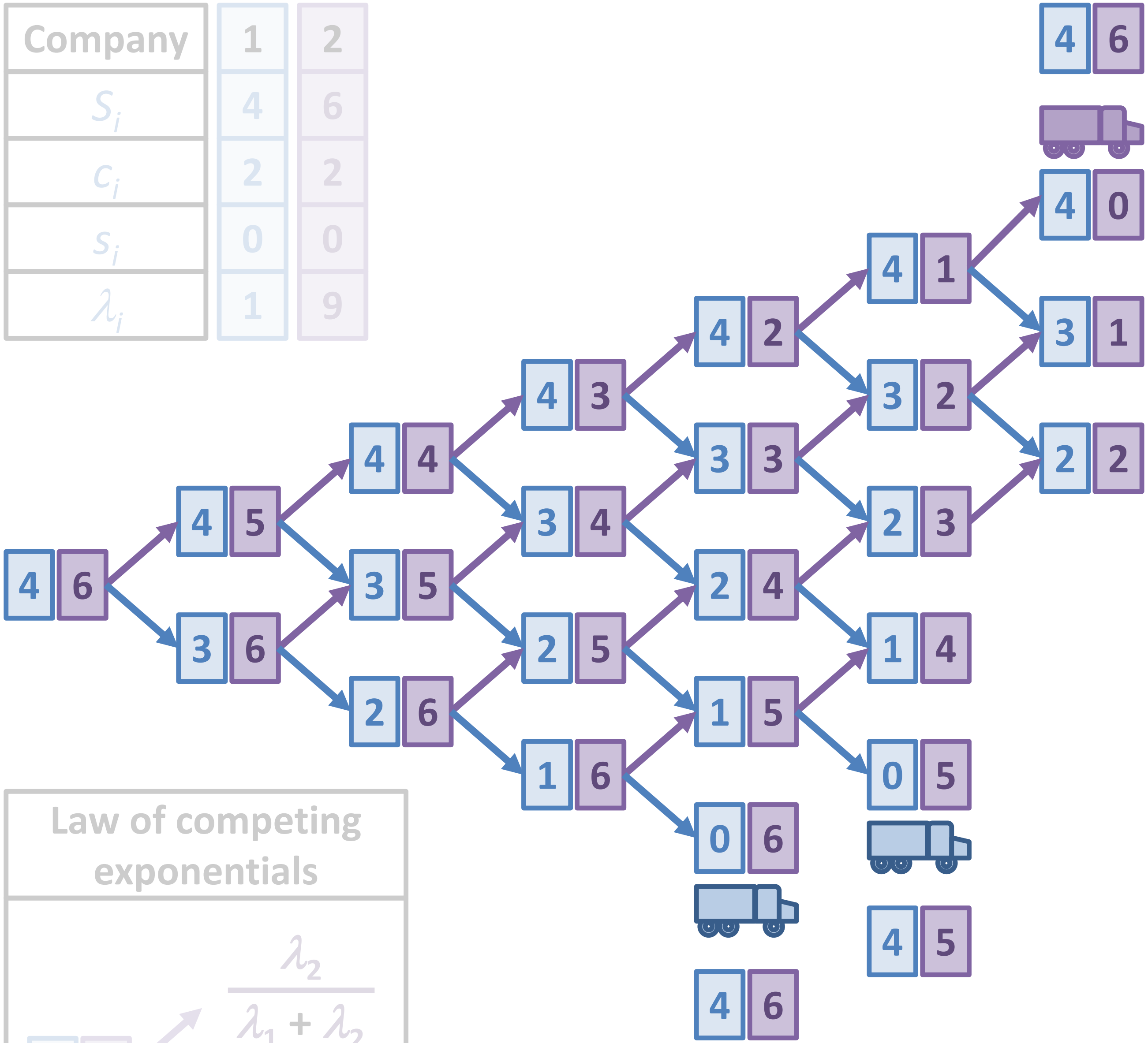


Company | 1 | 2
S_i | 4 | 6
c_i | 2 | 2
s_i | 0 | 0
λ_i | 1 | 9
Law of competing exponentials
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$
x y



Company
1
2
S_i
4
6
c_i
2
2
s_i
0
0
λ_i
1
9
Law of competing exponentials
x
y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$

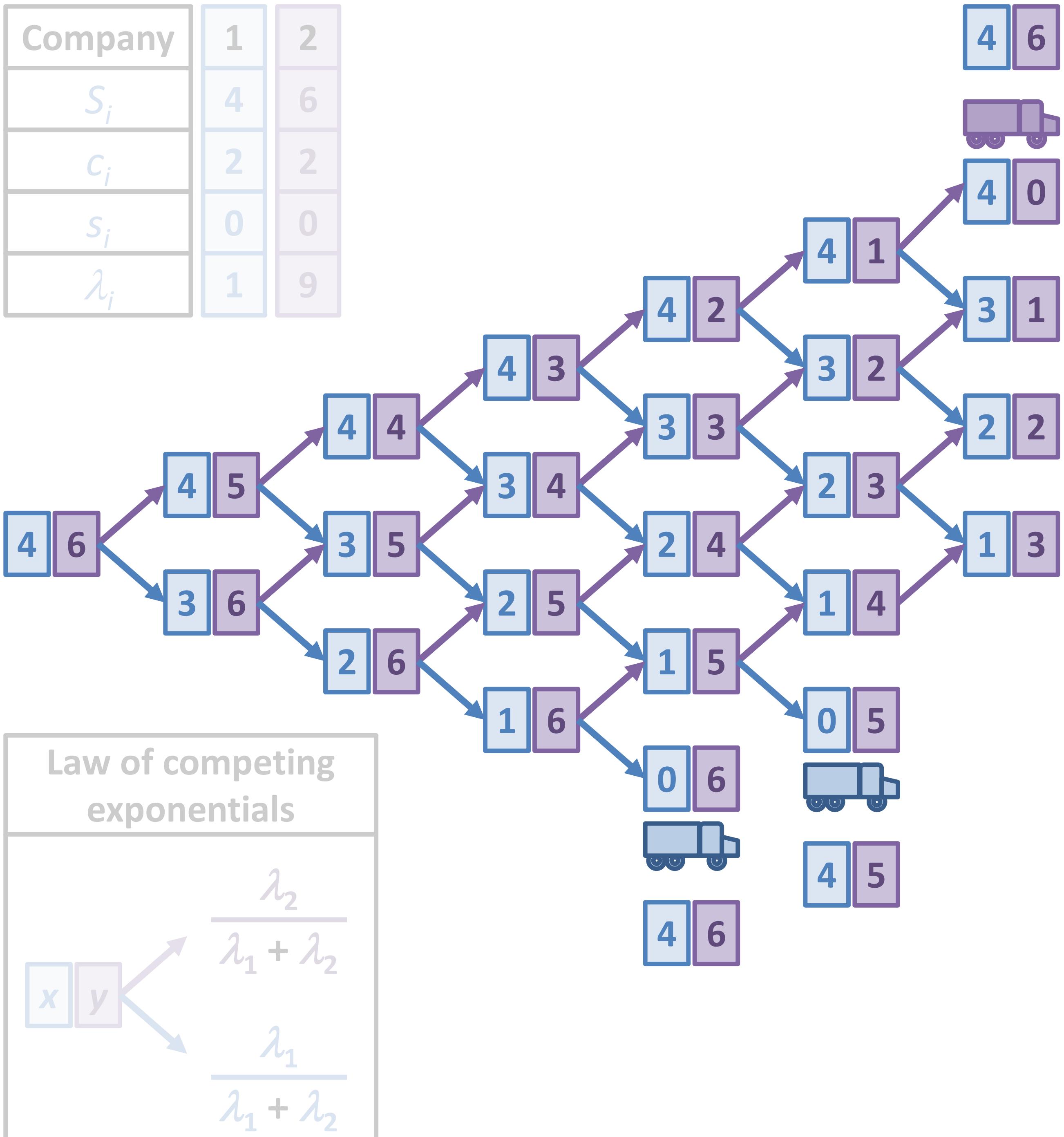
Company	1	2
S_i	4	6
c_i	2	2
s_i	0	0
λ_i	1	9

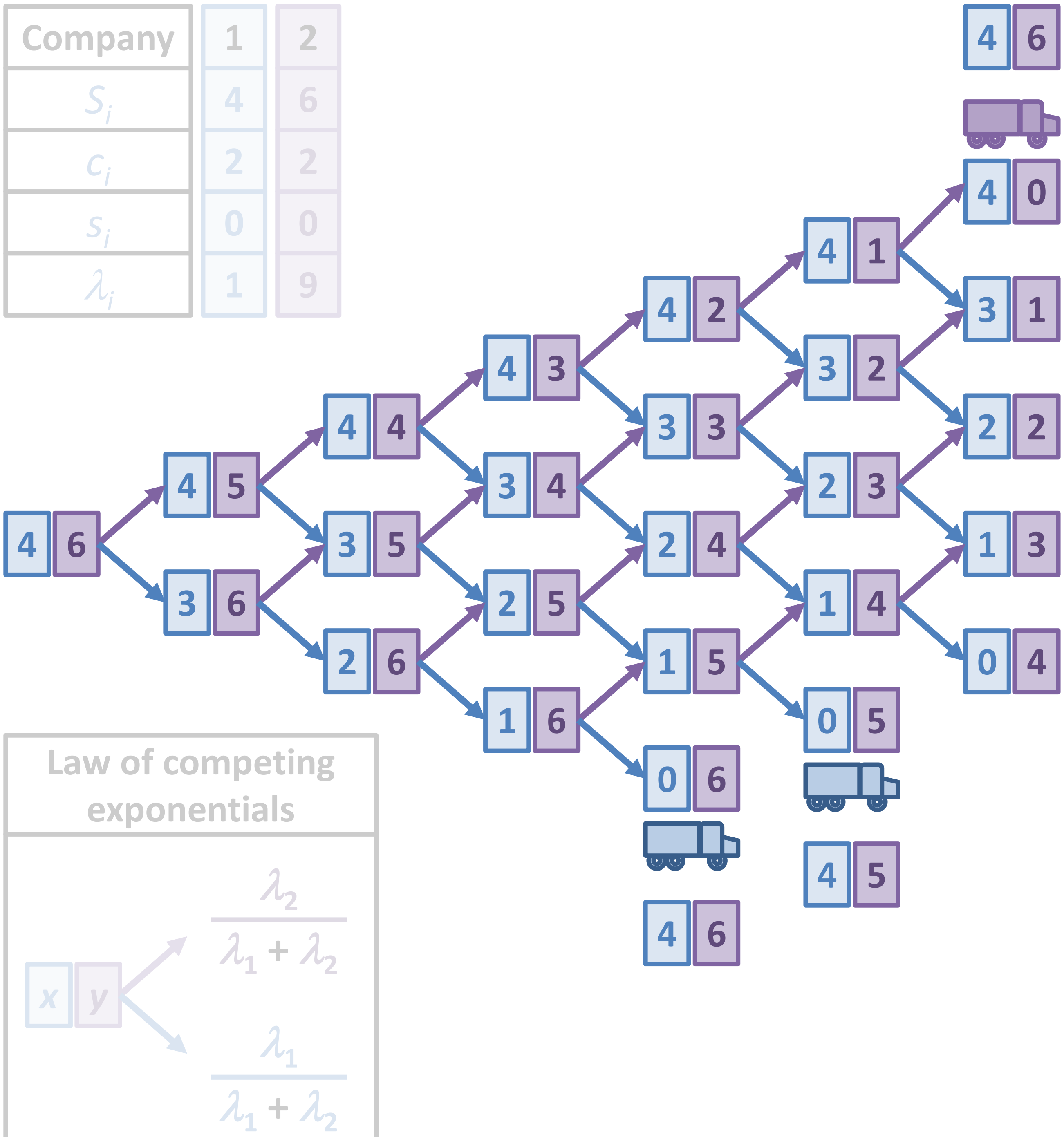


Law of competing exponentials

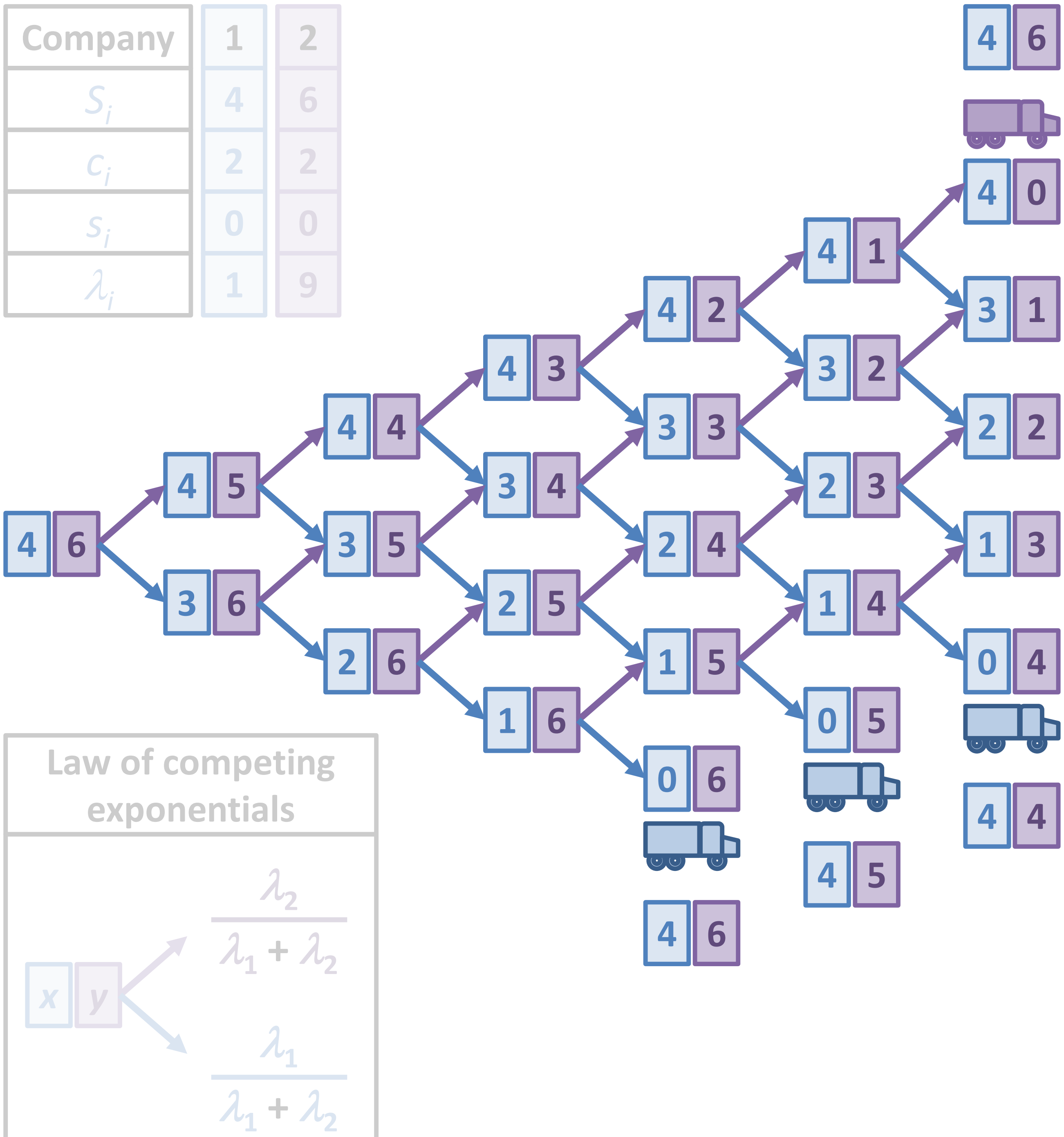
$$\frac{\lambda_2}{\lambda_1 + \lambda_2}$$

$$\frac{\lambda_1}{\lambda_1 + \lambda_2}$$


Company 1 2
S_i 4 6
c_i 2 2
s_i 0 0
λ_i 1 9
Law of competing exponentials
x y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$

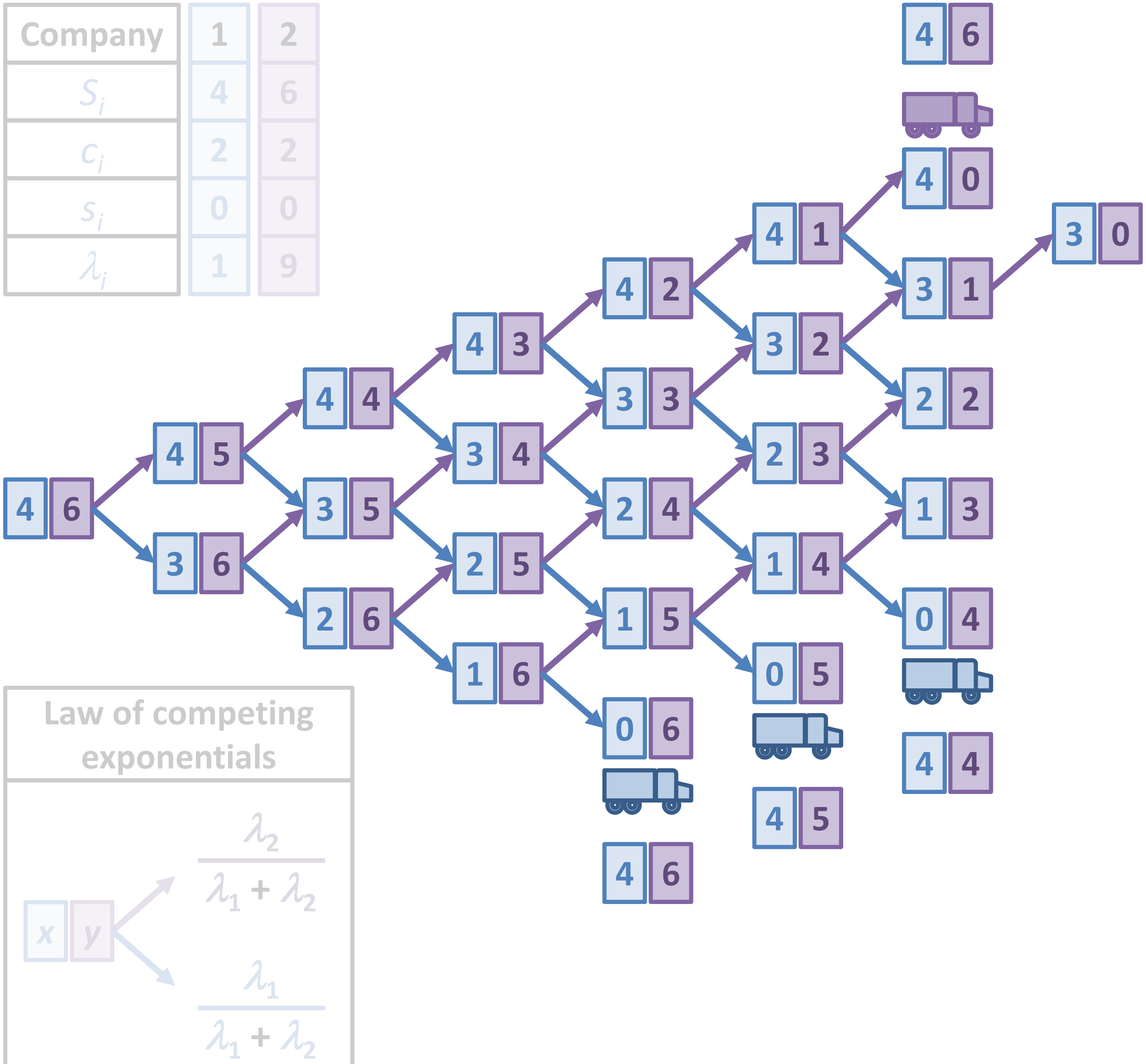


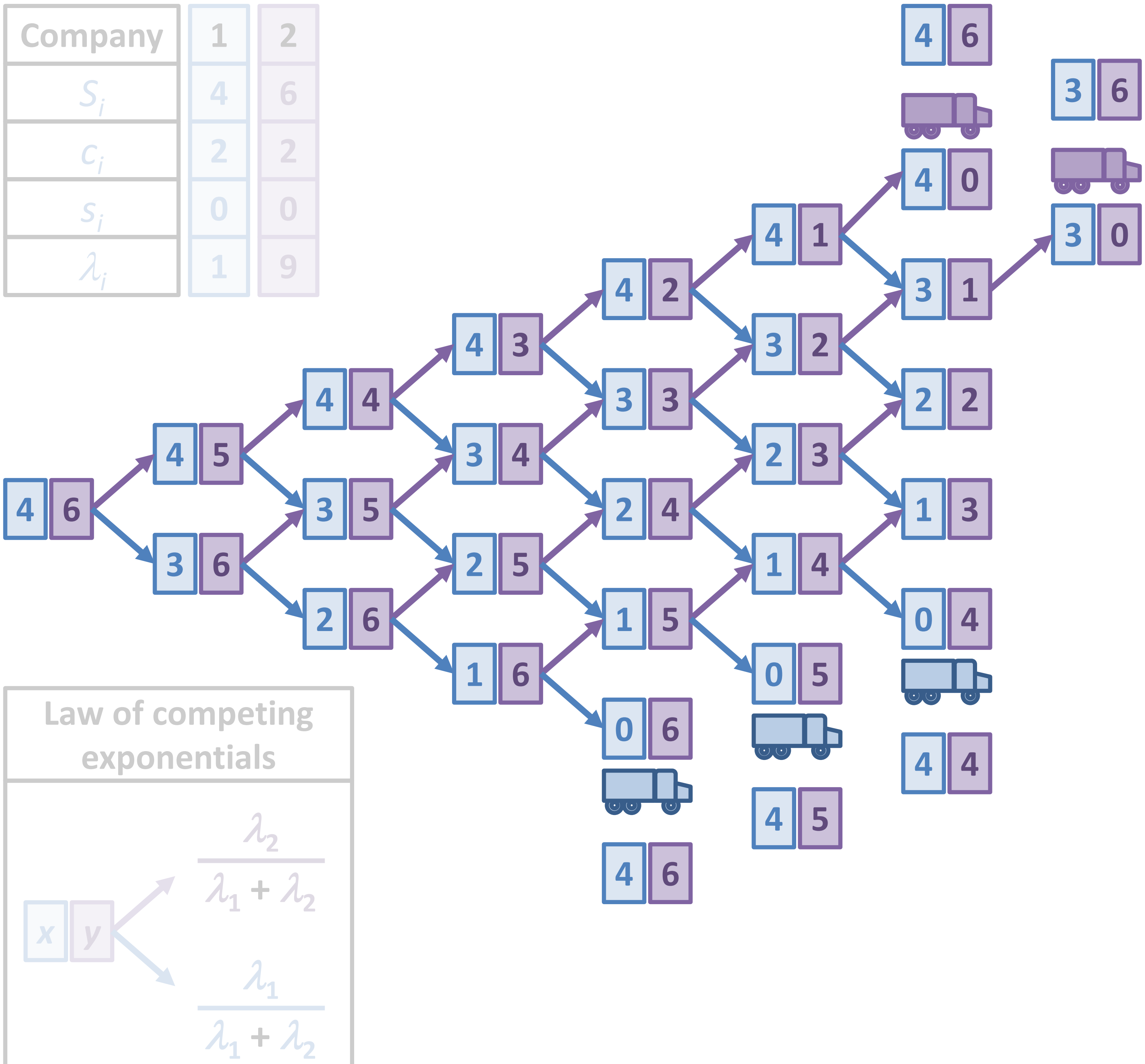
Company 1 2
S_i 4 6
c_i 2 2
s_i 0 0
λ_i 1 9
Law of competing exponentials
x y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$



Company
1
2
S_i
4
6
c_i
2
2
s_i
0
0
λ_i
1
9
Law of competing exponentials
x
y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$

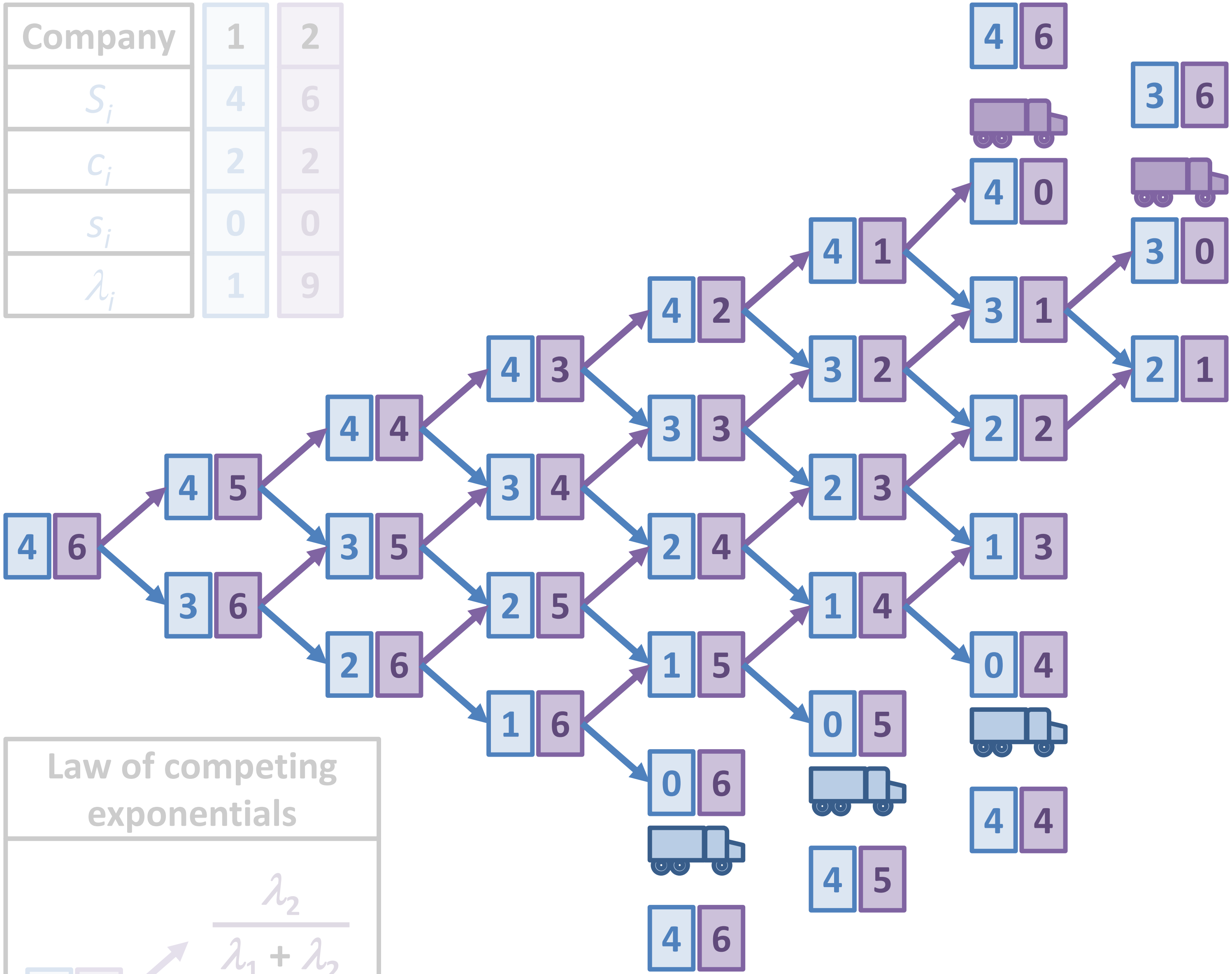
Company	1	2
S_i	4	6
c_i	2	2
s_i	0	0
λ_i	1	9





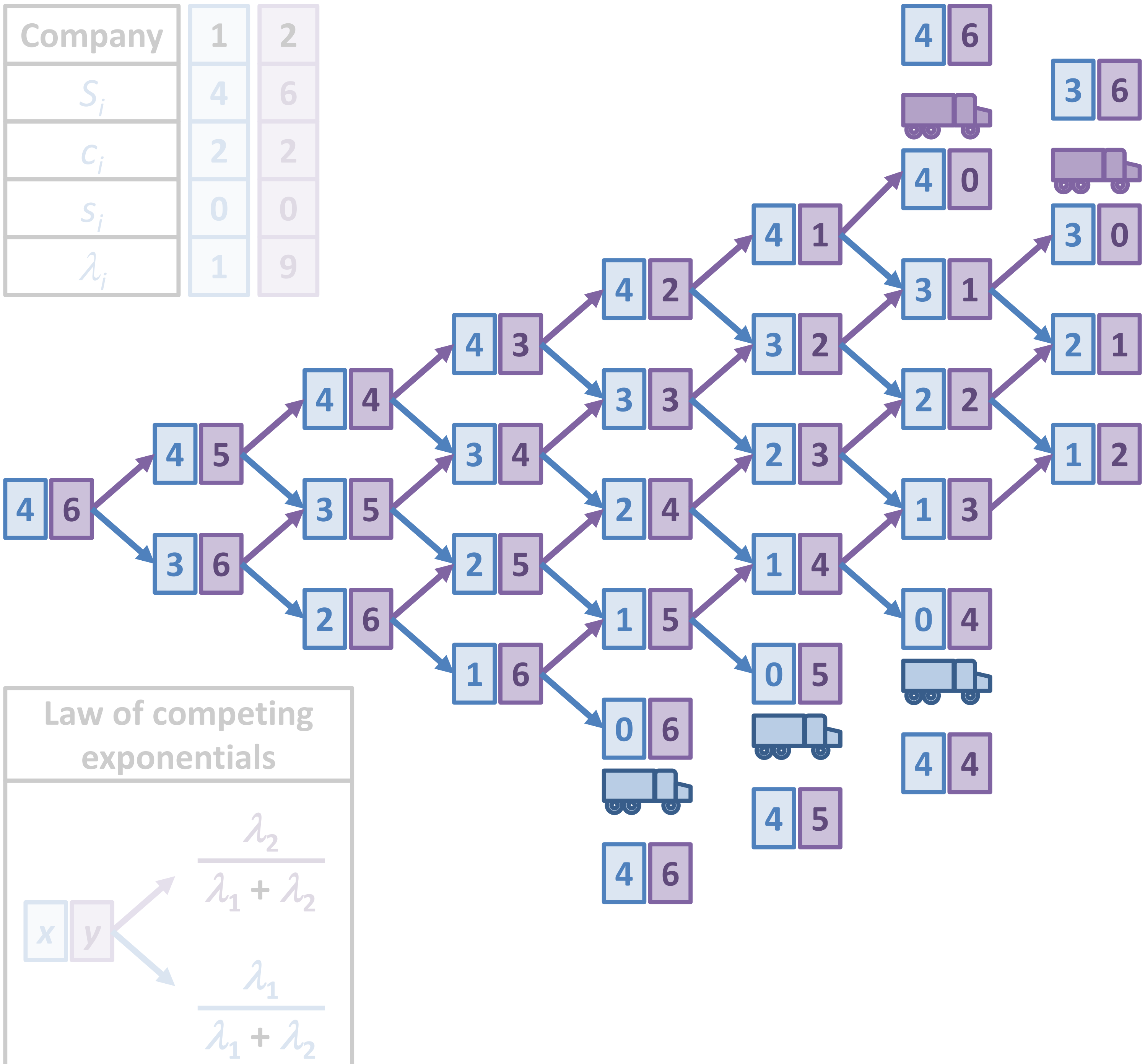
Company 1 2
S_i 4 6
c_i 2 2
s_i 0 0
λ_i 1 9
Law of competing exponentials
x y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$

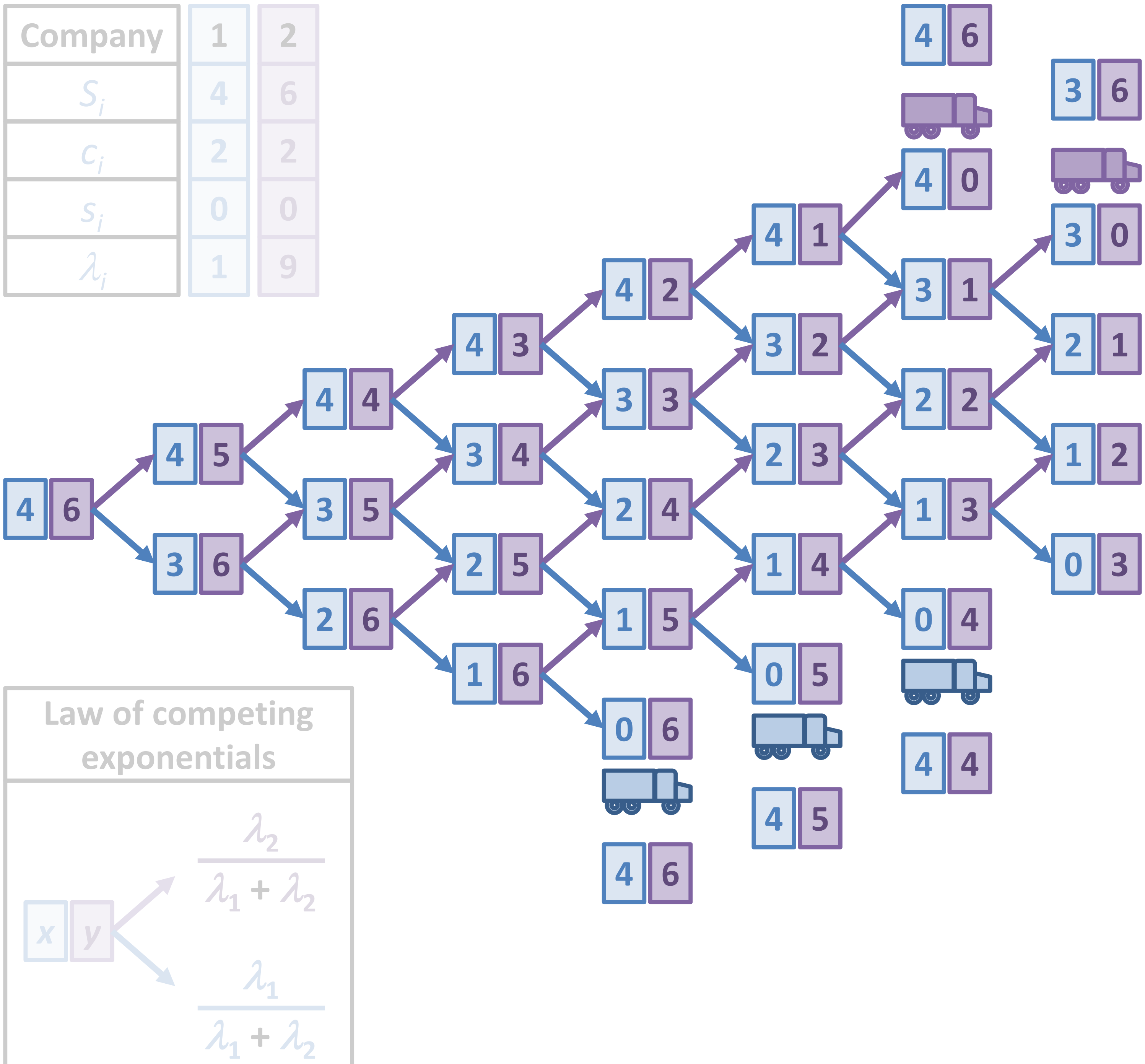
Company	1	2
S_i	4	6
c_i	2	2
s_i	0	0
λ_i	1	9

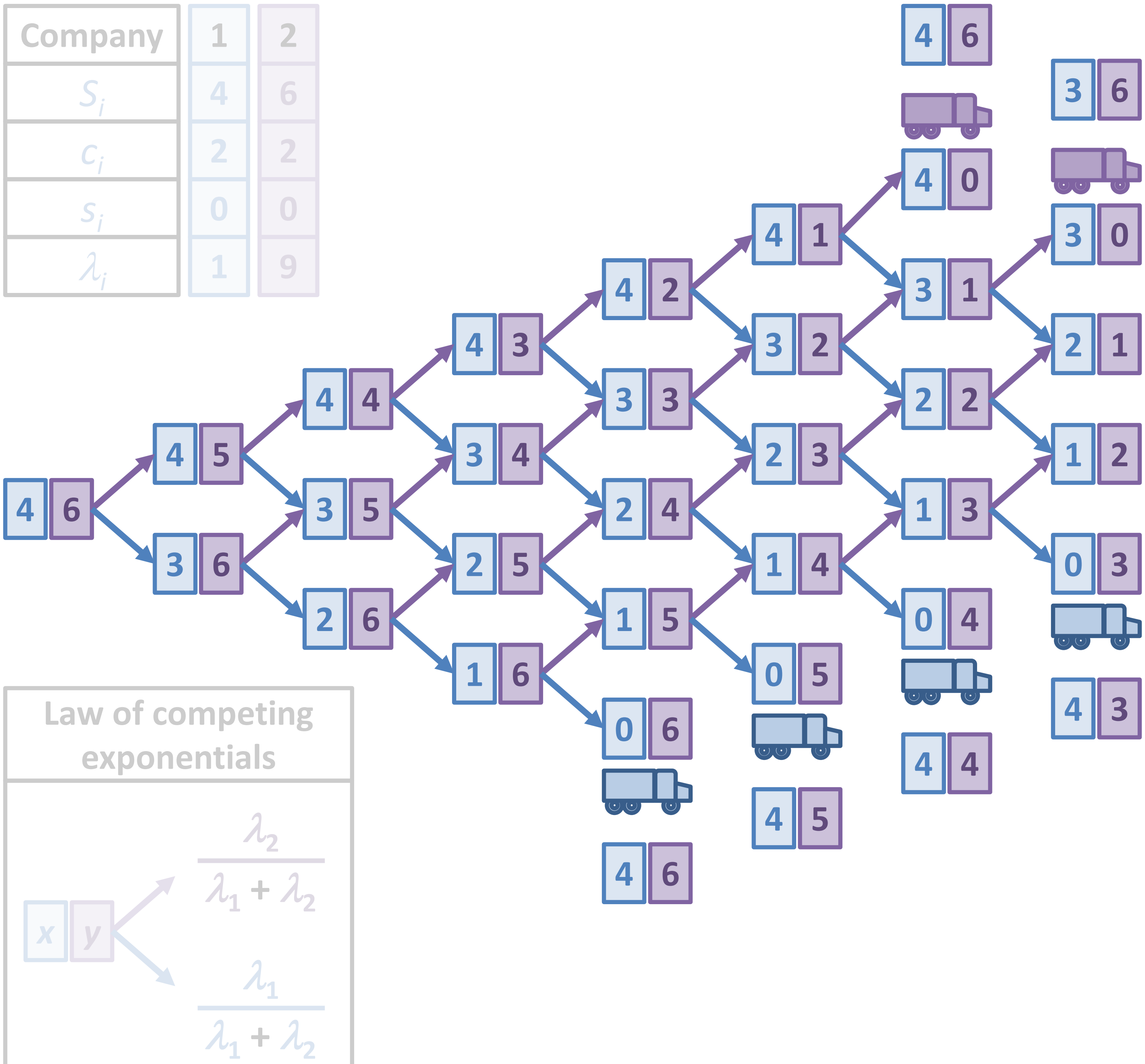


Law of competing exponentials

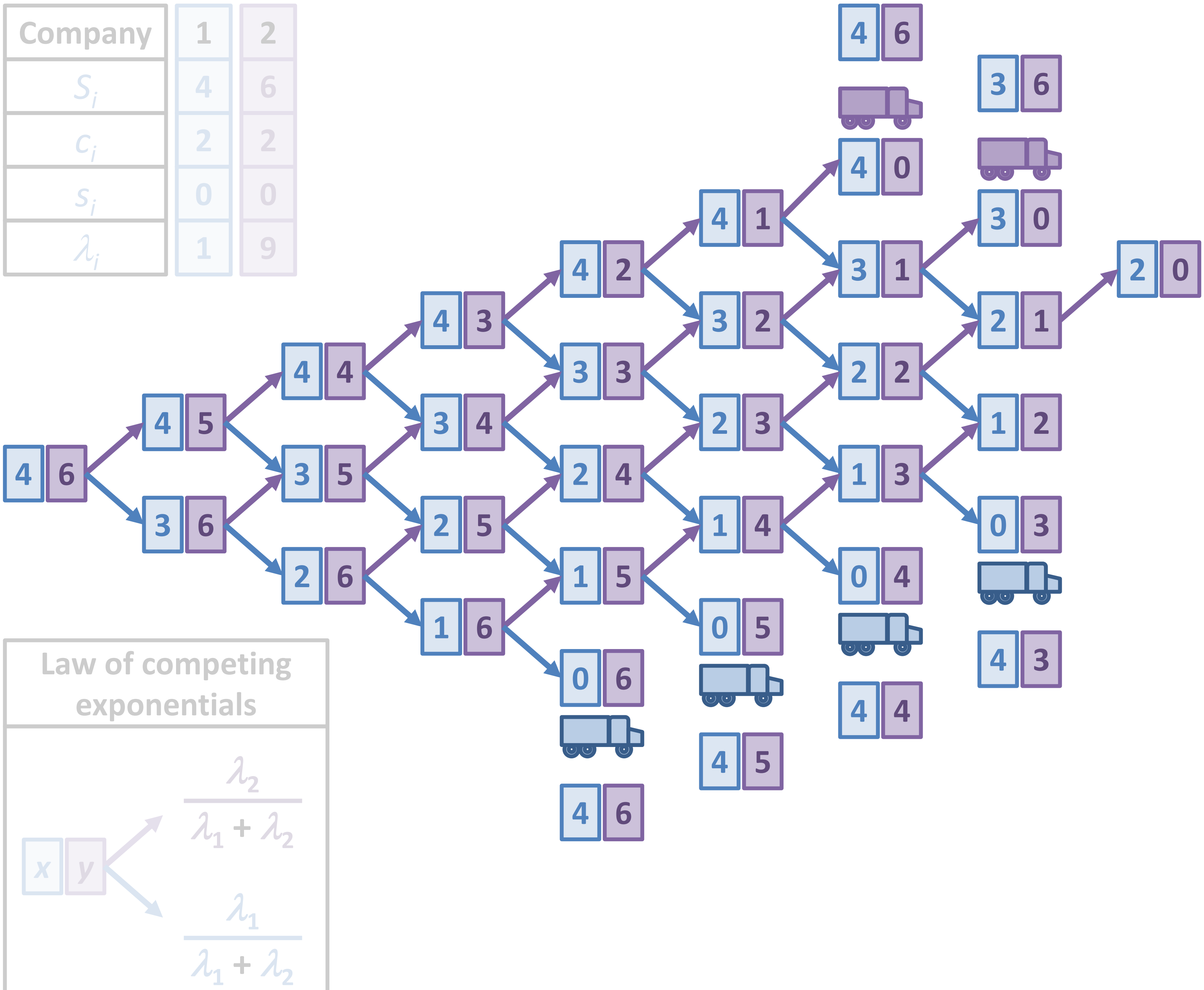
Company	1	2
S_i	4	6
c_i	2	2
s_i	0	0
λ_i	1	9




Company
1
2
S_i
4
6
c_i
2
2
s_i
0
0
λ_i
1
9
Law of competing exponentials
x
y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$



Company 1 2
S_i 4 6
c_i 2 2
s_i 0 0
λ_i 1 9
Law of competing exponentials
x y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$


Company 1 2
S_i 4 6
c_i 2 2
s_i 0 0
λ_i 1 9
Law of competing exponentials
x y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$
4 6
4 5
3 6
4 4
3 5
2 6
4 3
3 4
2 5
1 6
4 2
3 3
2 4
1 5
0 6
4 6
4 1
3 2
2 3
1 4
0 5
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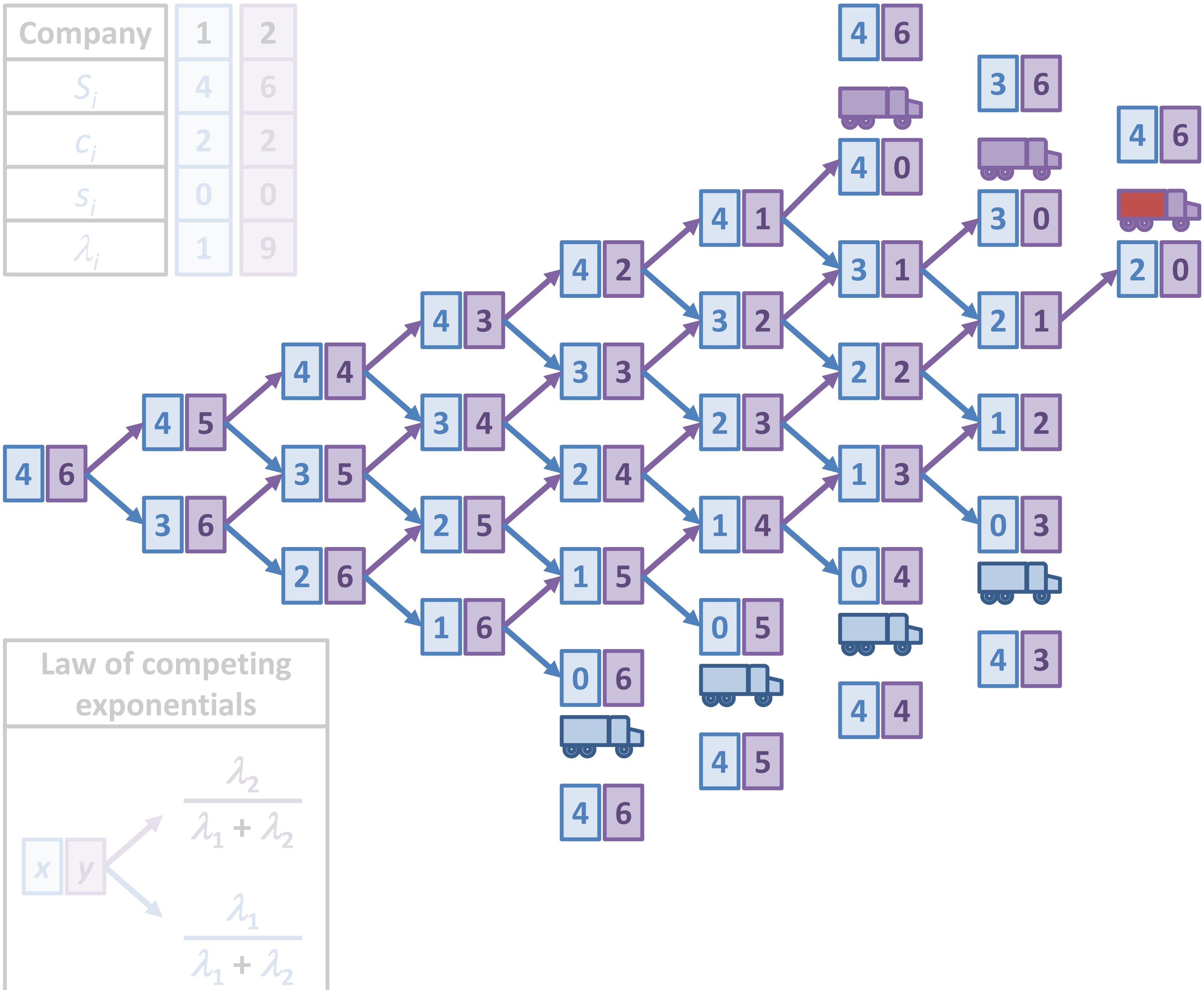
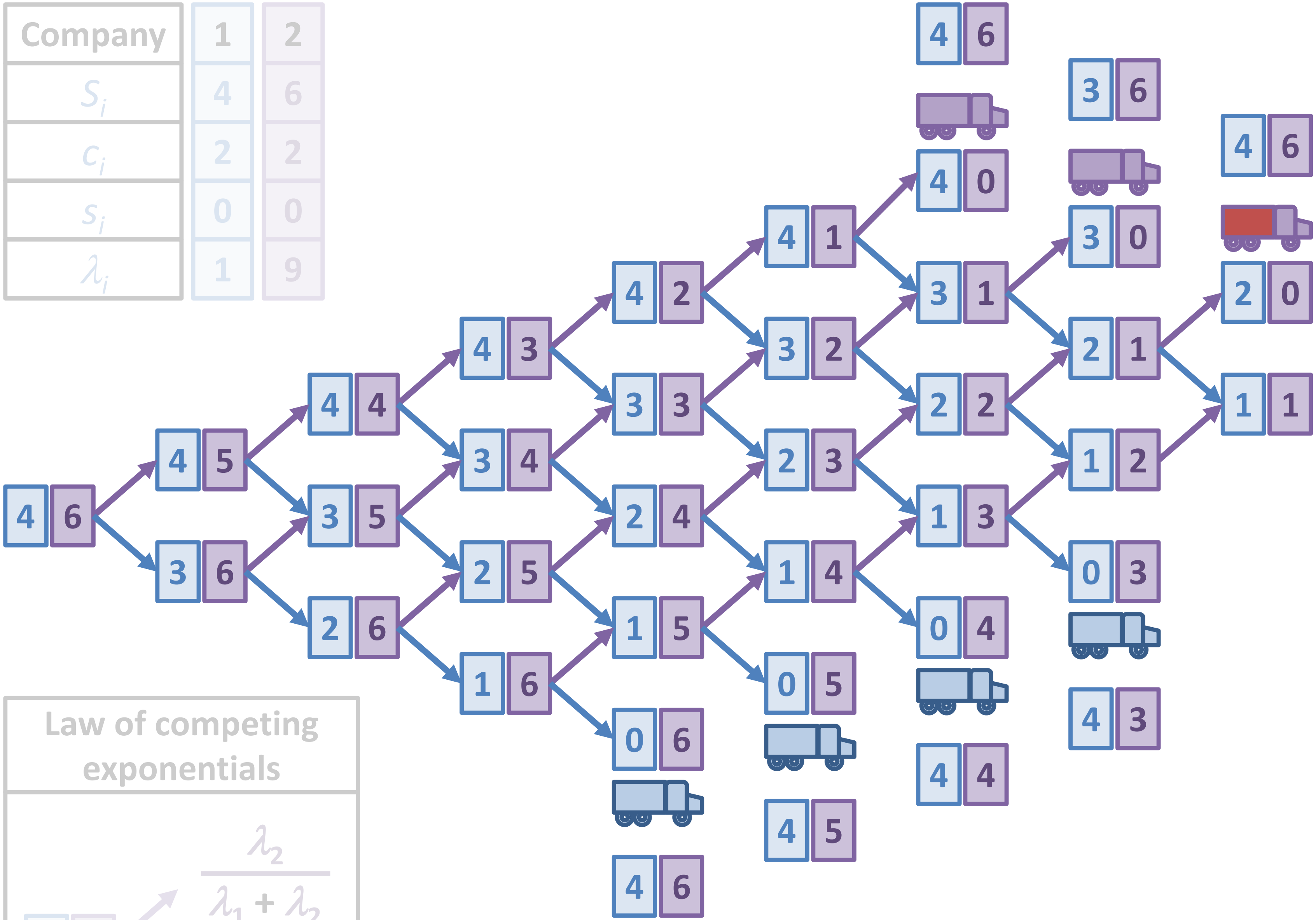
1 2
0 3
4 3
2 0



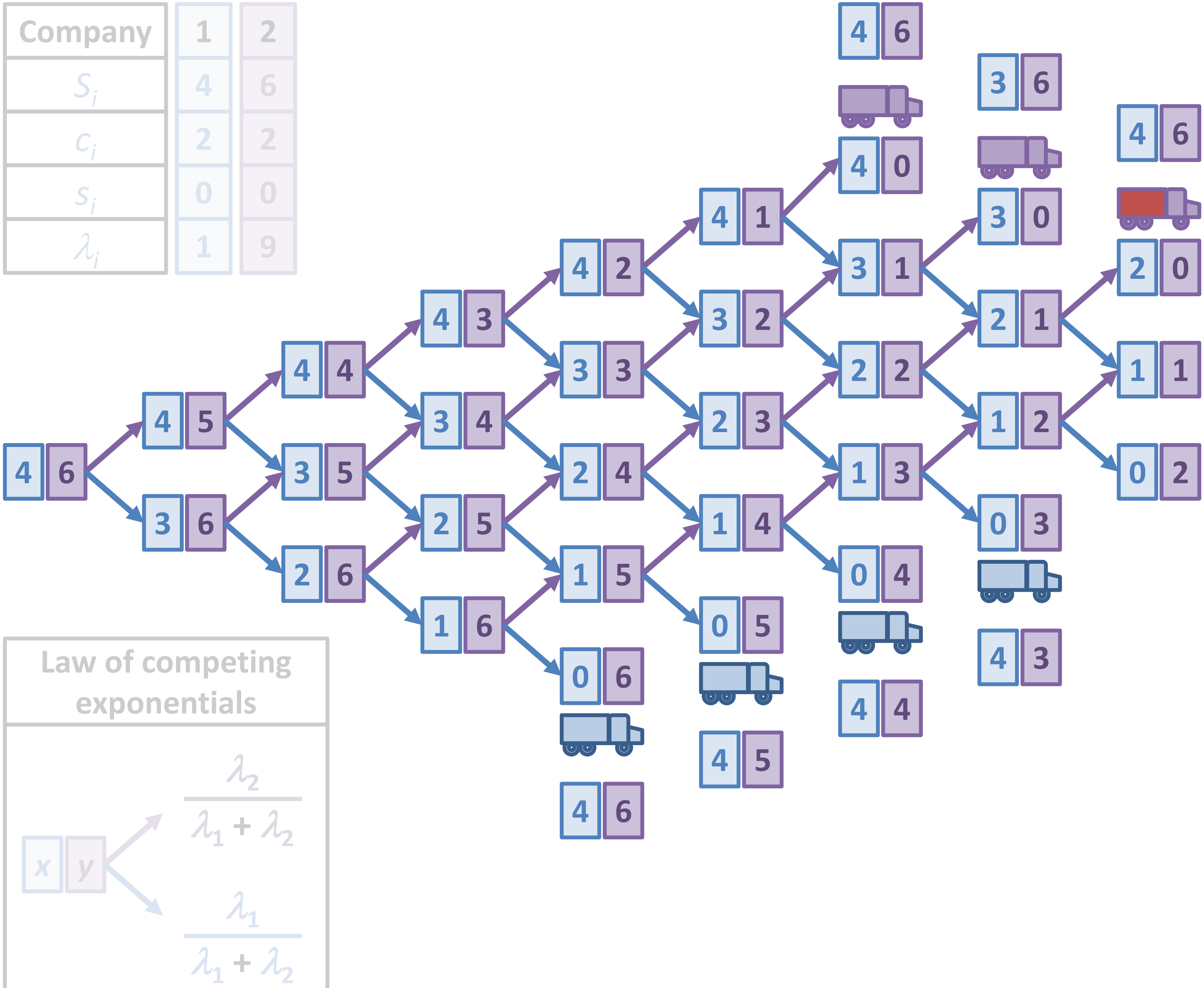
Company 1 2
S_i 4 6
c_i 2 2
s_i 0 0
λ_i 1 9
Law of competing exponentials
x y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$

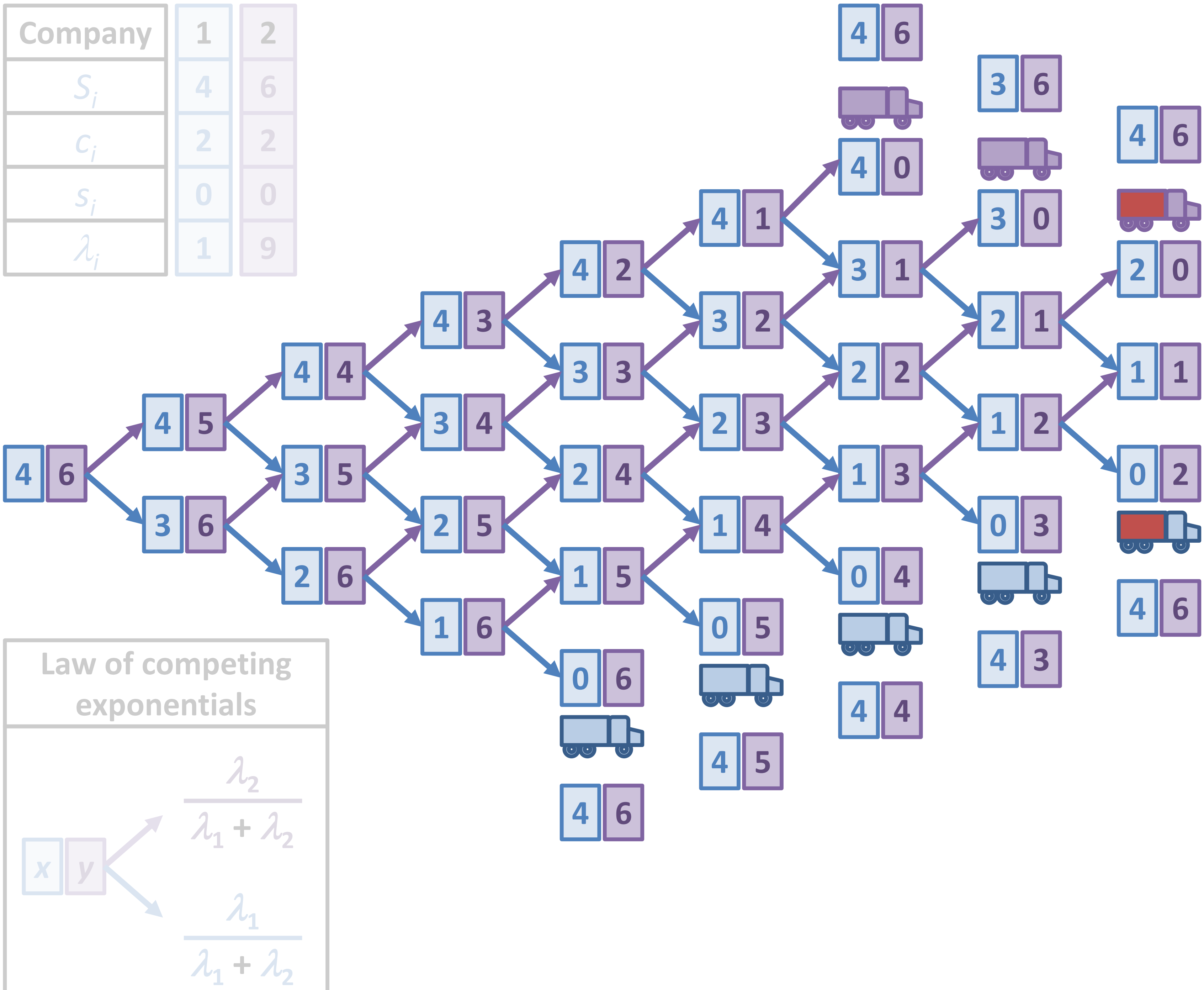
Company	1	2
S_i	4	6
c_i	2	2
s_i	0	0
λ_i	1	9



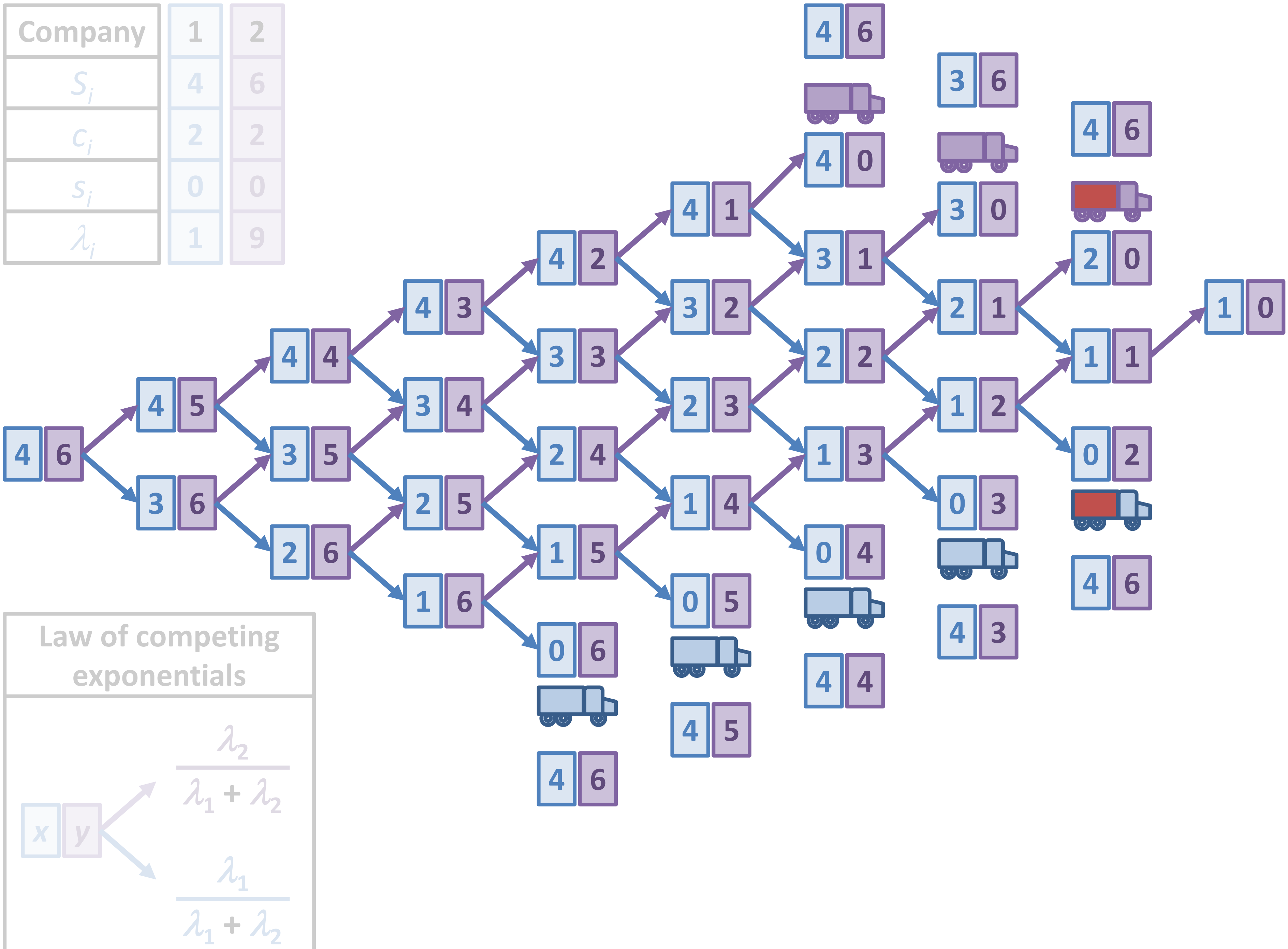
Law of competing exponentials

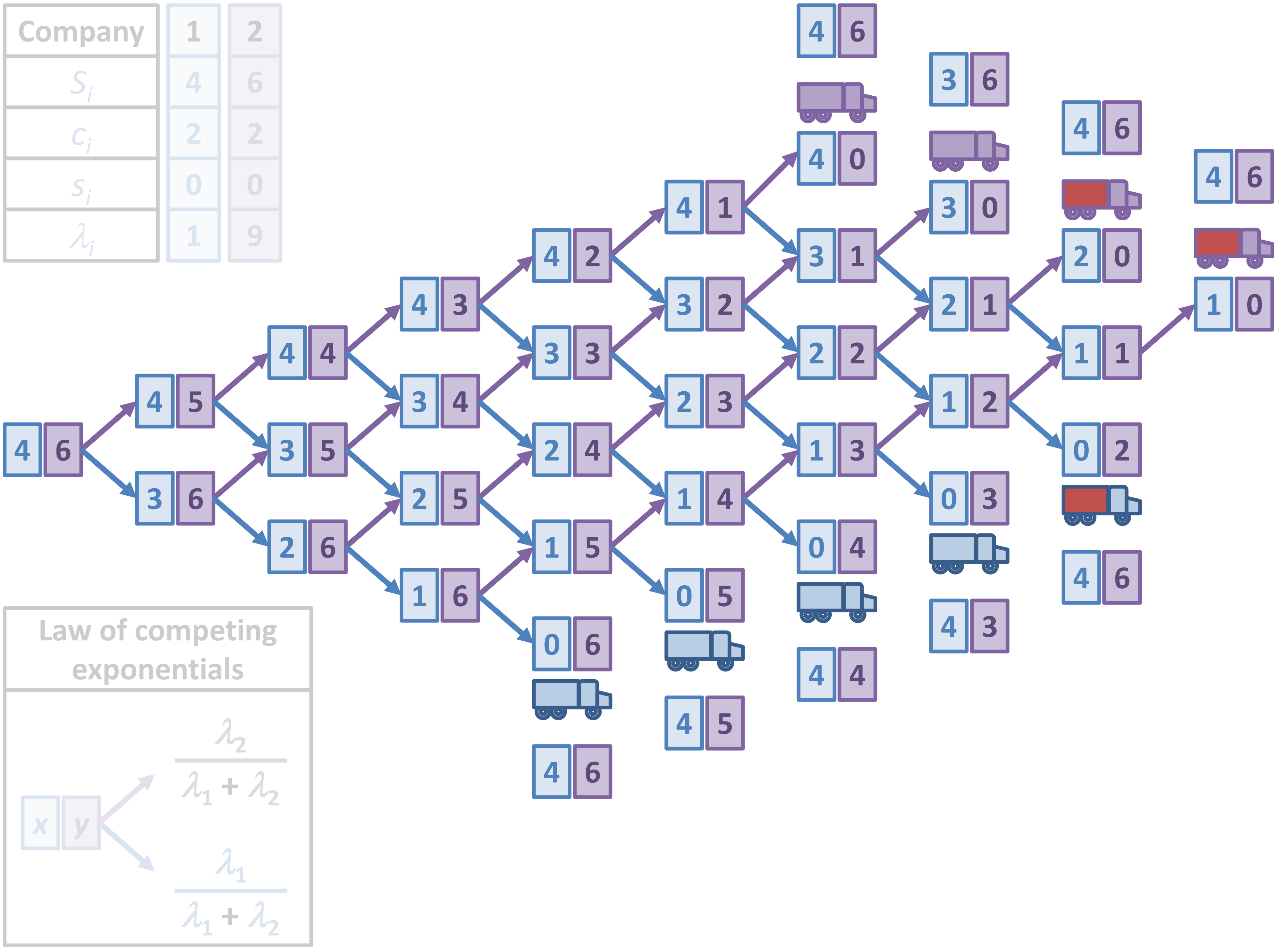
Company	1	2
S_i	4	6
c_i	2	2
s_i	0	0
λ_i	1	9



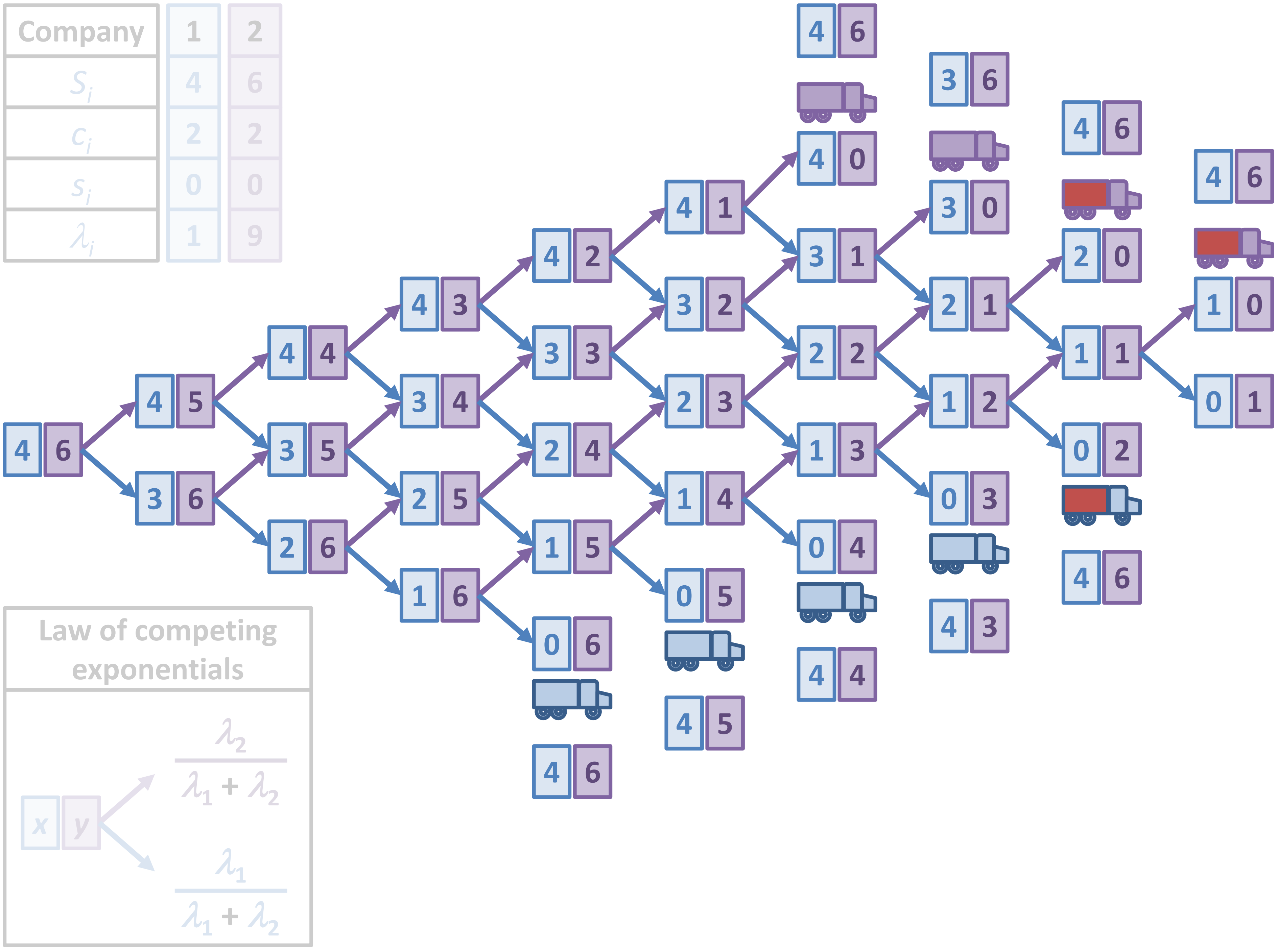


Company 1 2
S_i 4 6
c_i 2 2
s_i 0 0
λ_i 1 9
Law of competing exponentials
x y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$

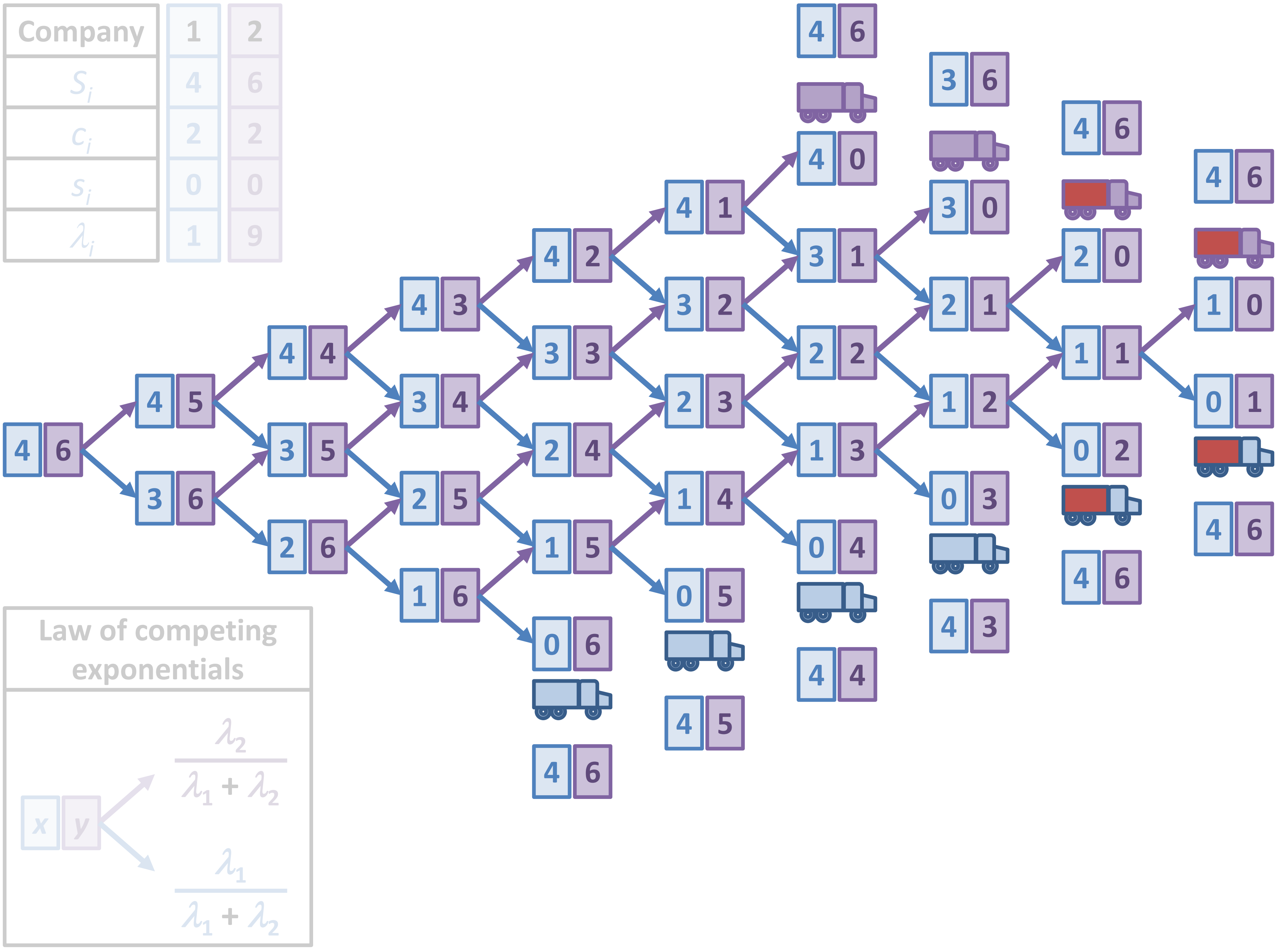

Company 1 2
S_i 4 6
c_i 2 2
s_i 0 0
λ_i 1 9
Law of competing exponentials
x y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$

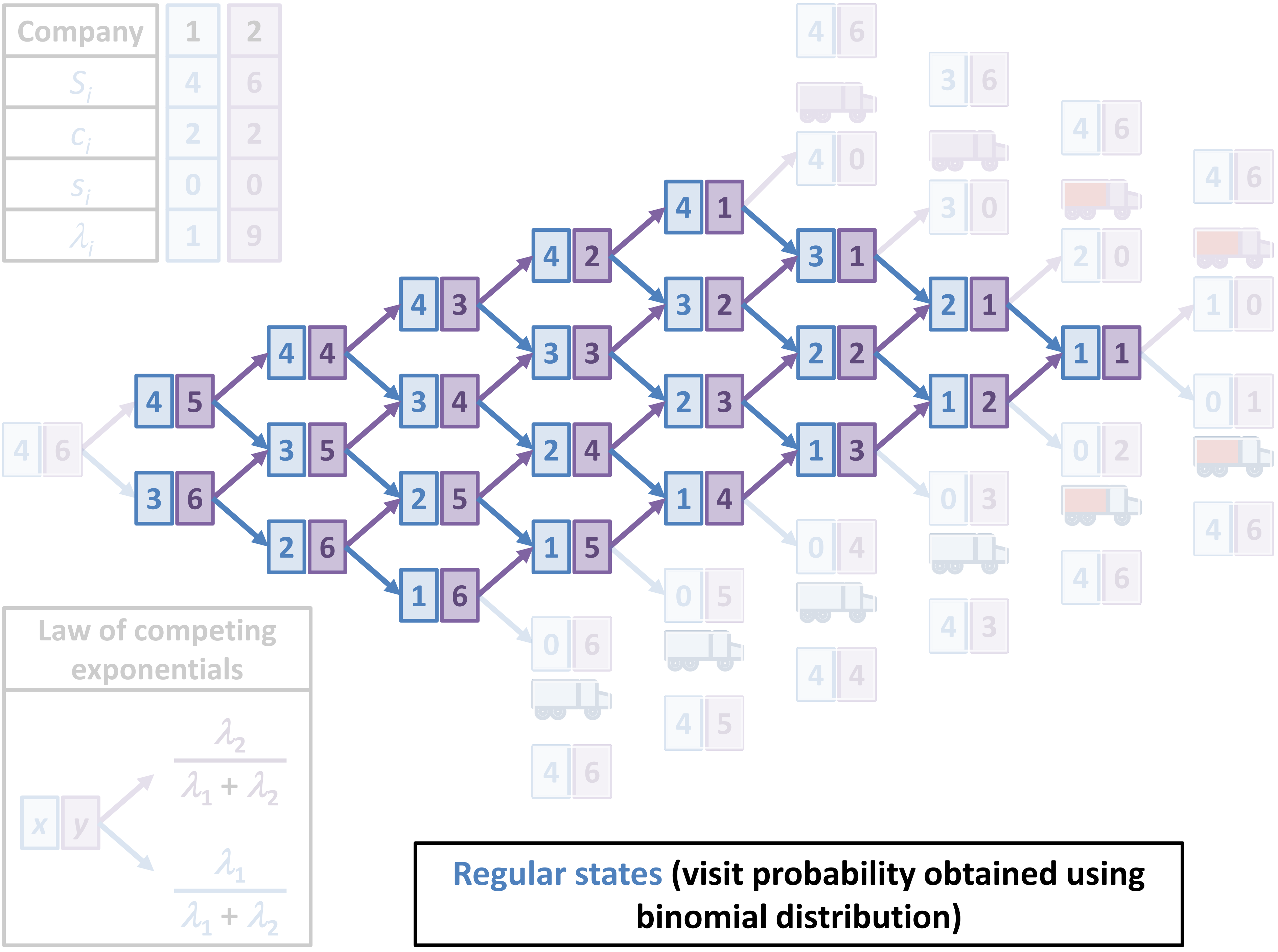


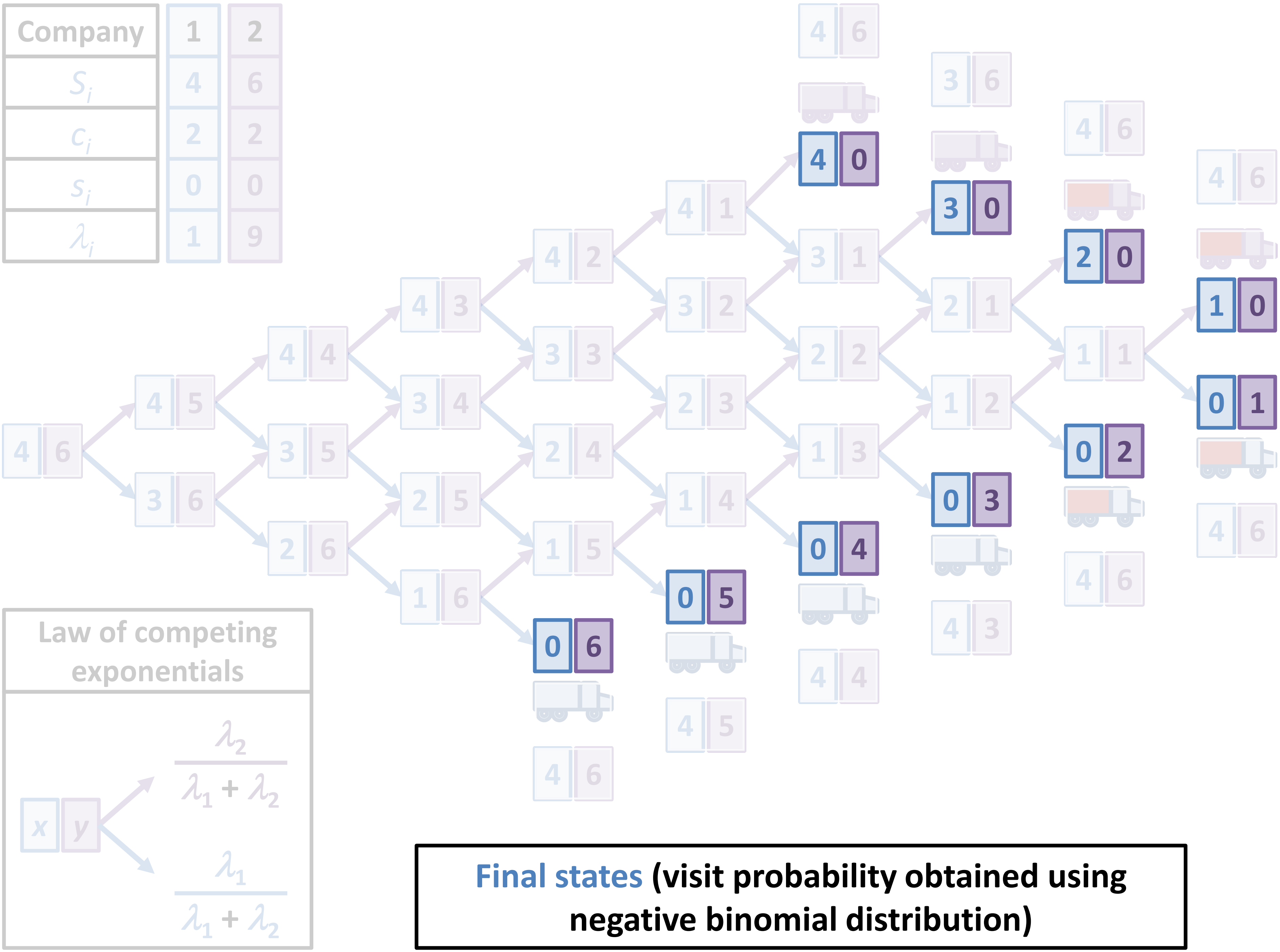
Company 1 2
S_i 4 6
c_i 2 2
s_i 0 0
λ_i 1 9
Law of competing exponentials
x y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$



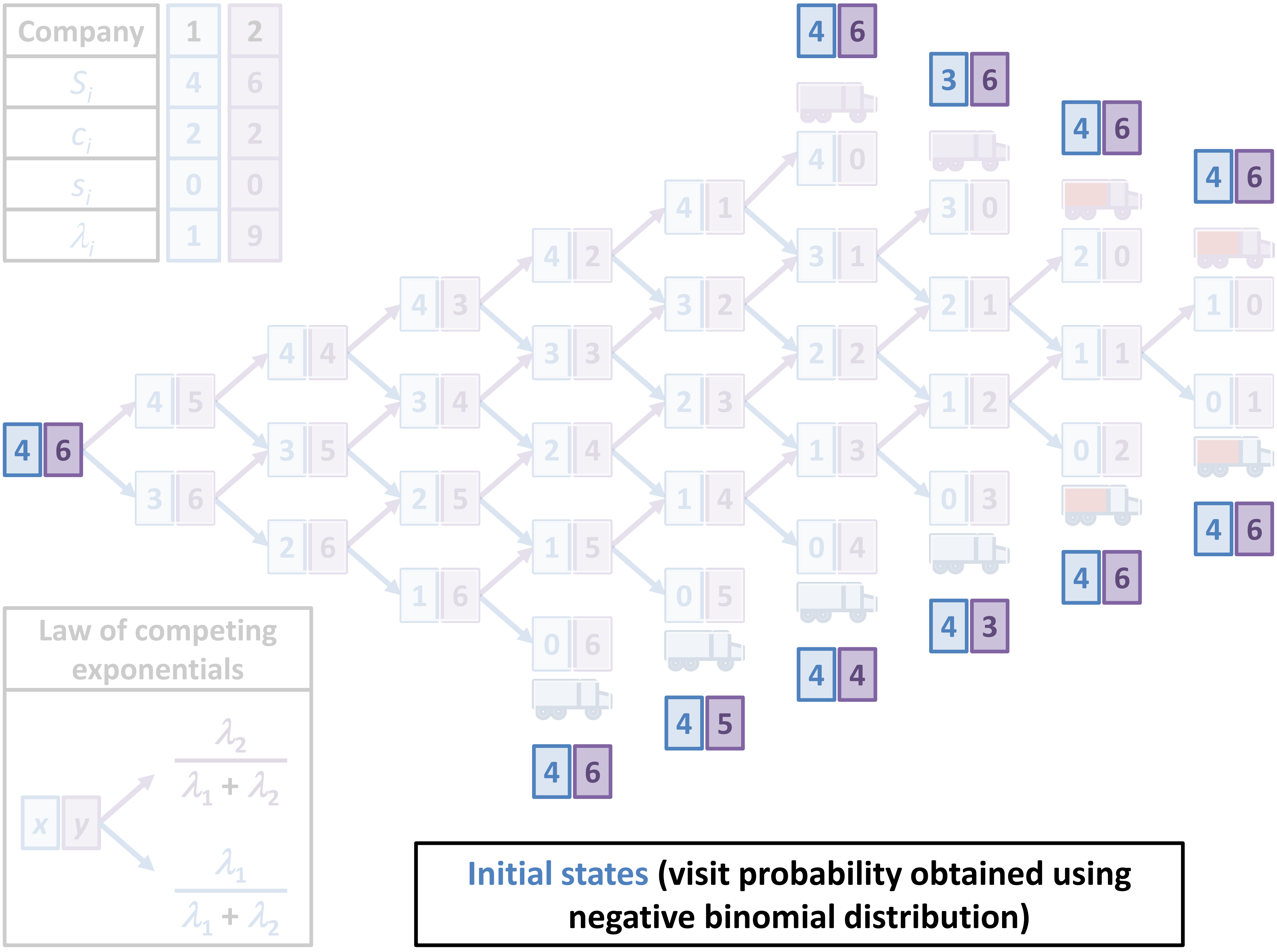
Company 1 2
S_i 4 6
c_i 2 2
s_i 0 0
λ_i 1 9
Law of competing exponentials
x y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$

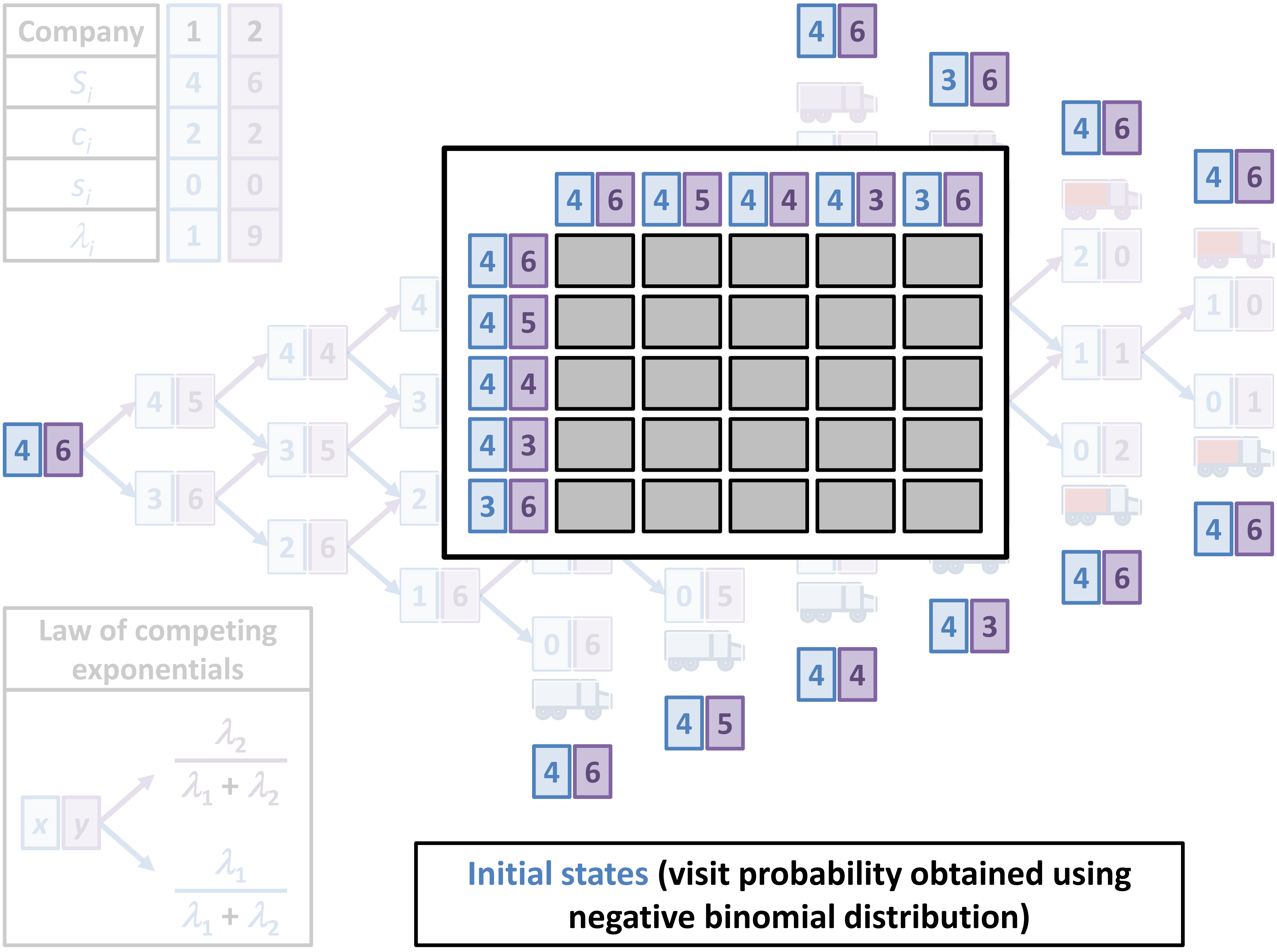

Company 1 2
S_i 4 6
c_i 2 2
s_i 0 0
λ_i 1 9
Law of competing exponentials
x y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$

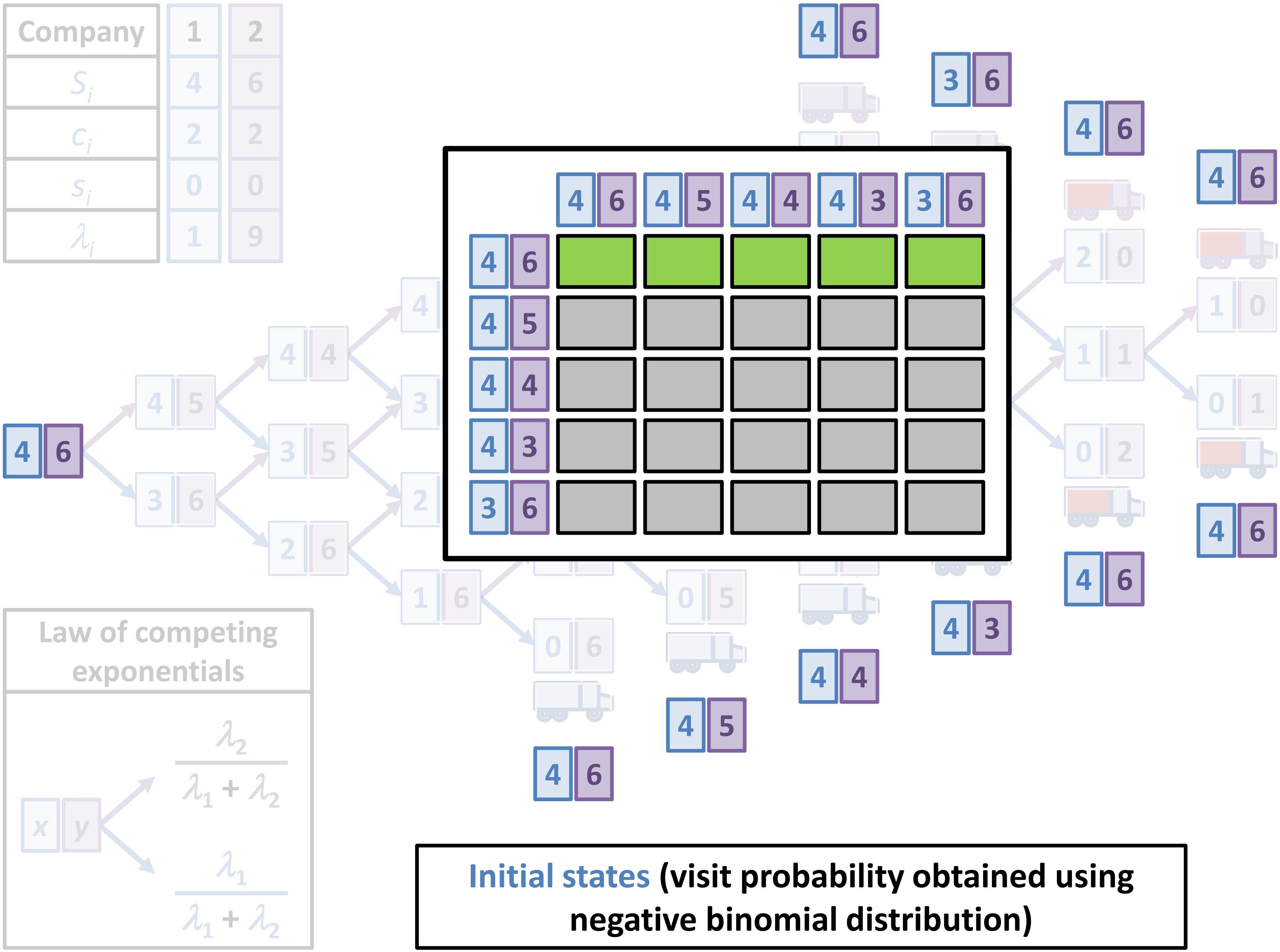

Company 1 2
S_i 4 6
c_i 2 2
s_i 0 0
λ_i 1 9
Law of competing exponentials
x y
λ2 / (λ1 + λ2)
λ1 / (λ1 + λ2)
Regular states (visit probability obtained using binomial distribution)



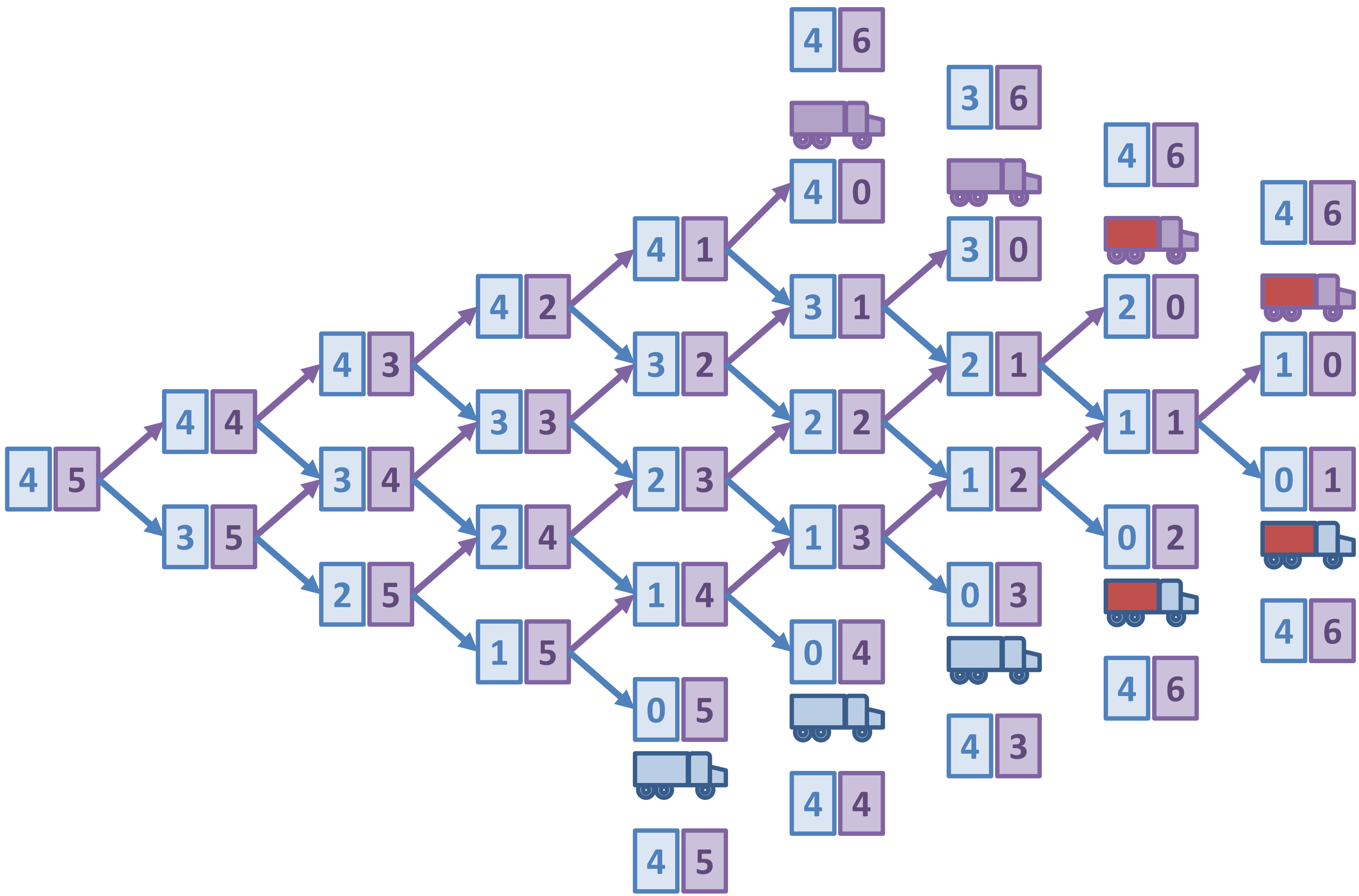
Company 1 2
S_i 4 6
c_i 2 2
s_i 0 0
λ_i 1 9
Law of competing exponentials
x y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$
Final states (visit probability obtained using negative binomial distribution)


Company 1 2
S_i 4 6
c_i 2 2
s_i 0 0
λ_i 1 9
Law of competing exponentials
x y
$\frac{\lambda_2}{\lambda_1+\lambda_2}$
$\frac{\lambda_1}{\lambda_1+\lambda_2}$
Initial states (visit probability obtained using negative binomial distribution)


Company
1
2
S_i
4
6
c_i
2
2
s_i
0
0
λ_i
1
9
Law of competing exponentials
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$
Initial states (visit probability obtained using negative binomial distribution)



Company
1
2
S_i
4
6
c_i
2
2
s_i
0
0
λ_i
1
9
Law of competing exponentials
x
y
$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
$\frac{\lambda_1}{\lambda_1 + \lambda_2}$
Initial states (visit probability obtained using negative binomial distribution)



4 5
4 4
3 5
4 3
3 4
2 5
4 2
3 3
2 4
1 5
4 1
3 2
2 3
1 4
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4 5
4 6
4 0
3 1
2 2
1 3
0 4
4 4
3 6
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2 0
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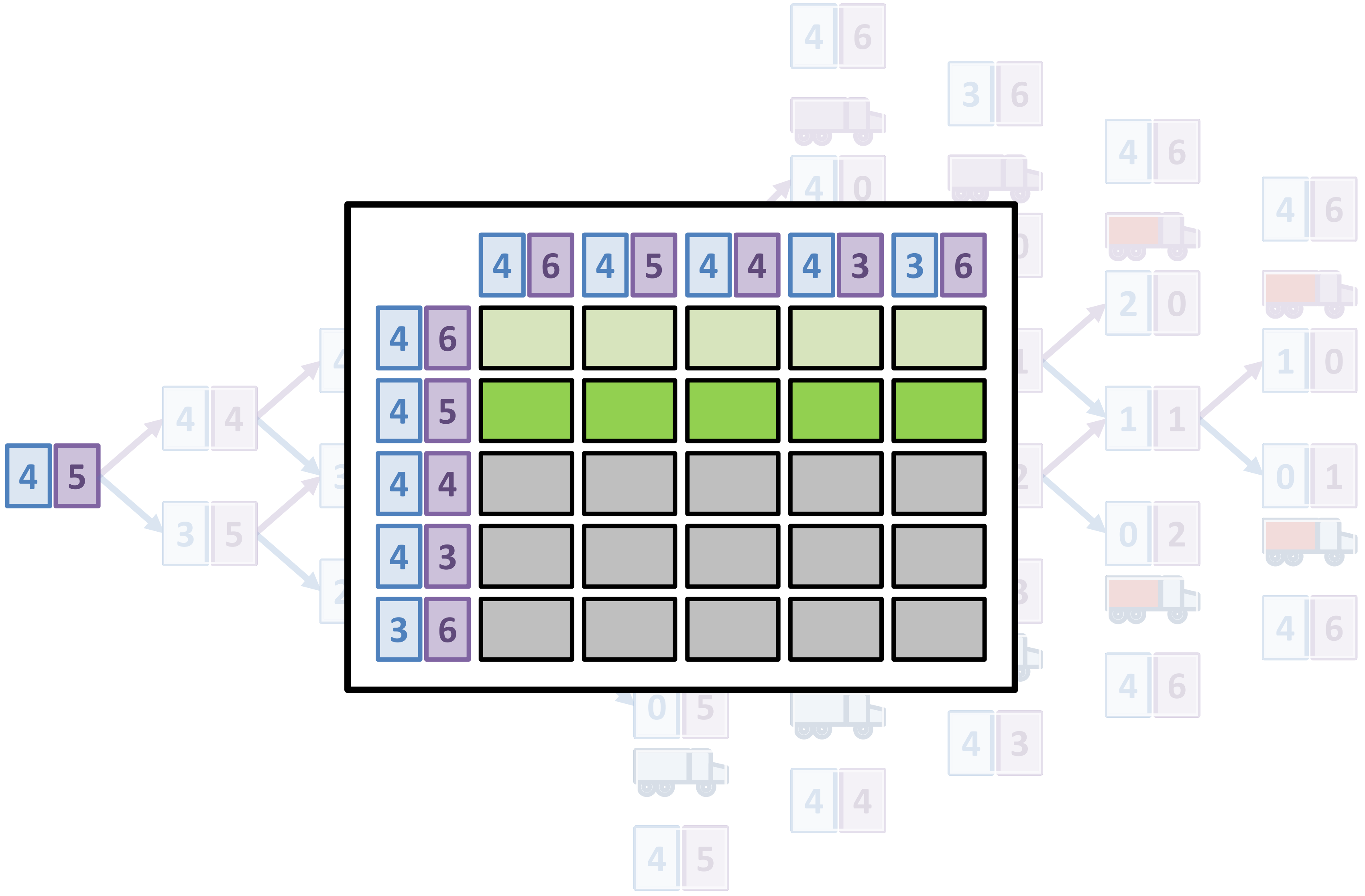
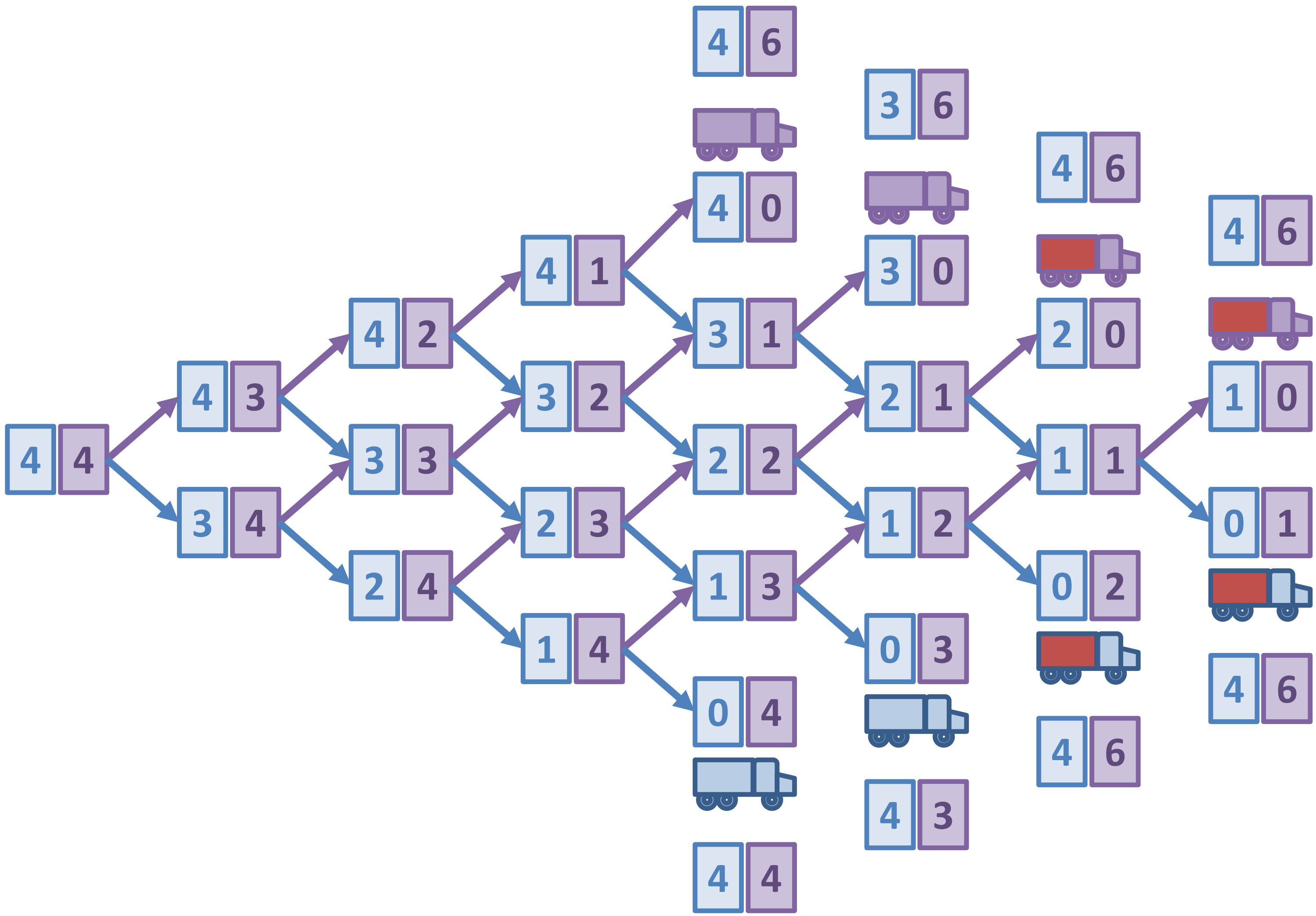
0 2
4 6
4 6
1 0
0 1
4 6





4 4
4 3
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1 4
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1 3
0 4
4 4
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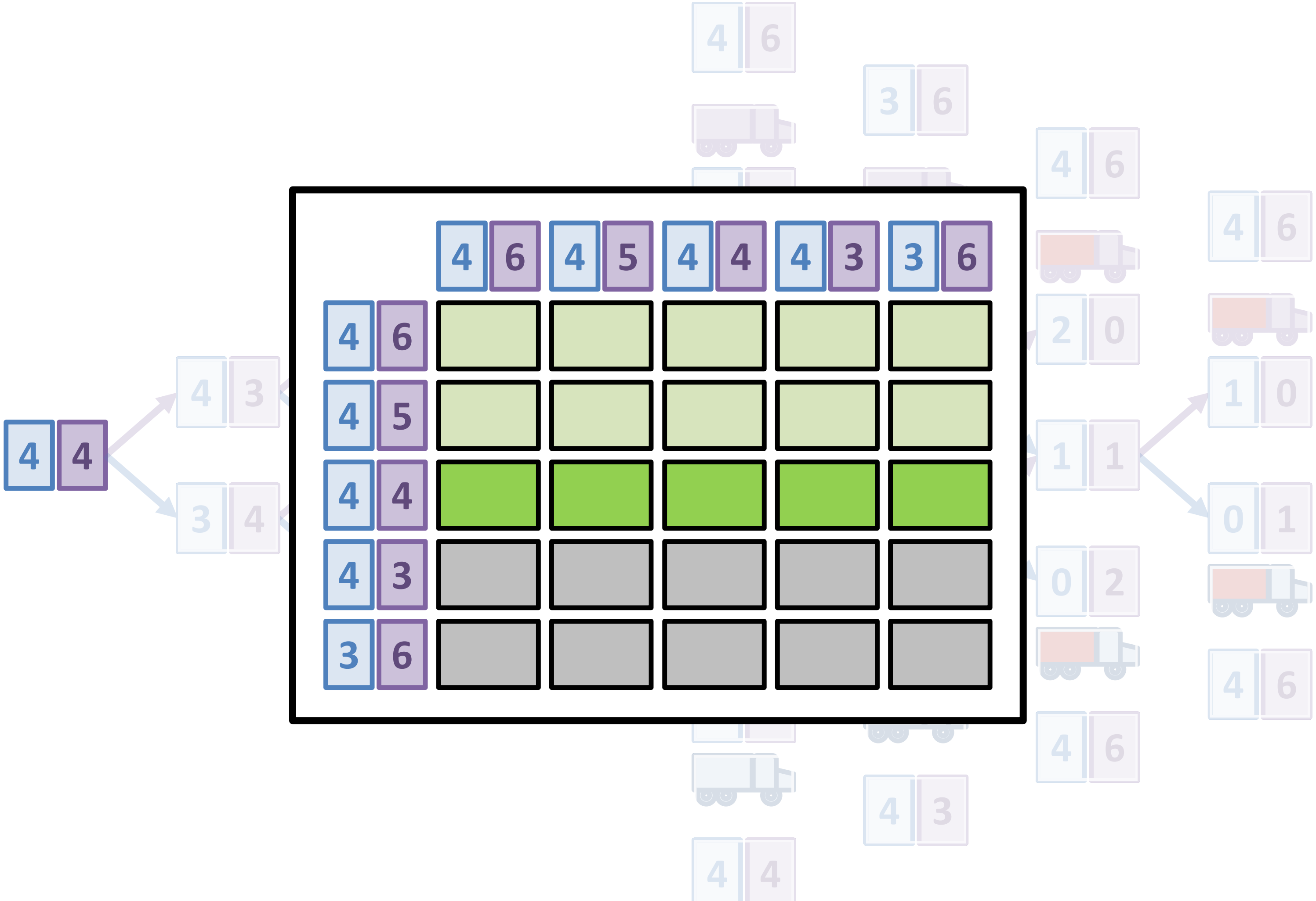
4 6



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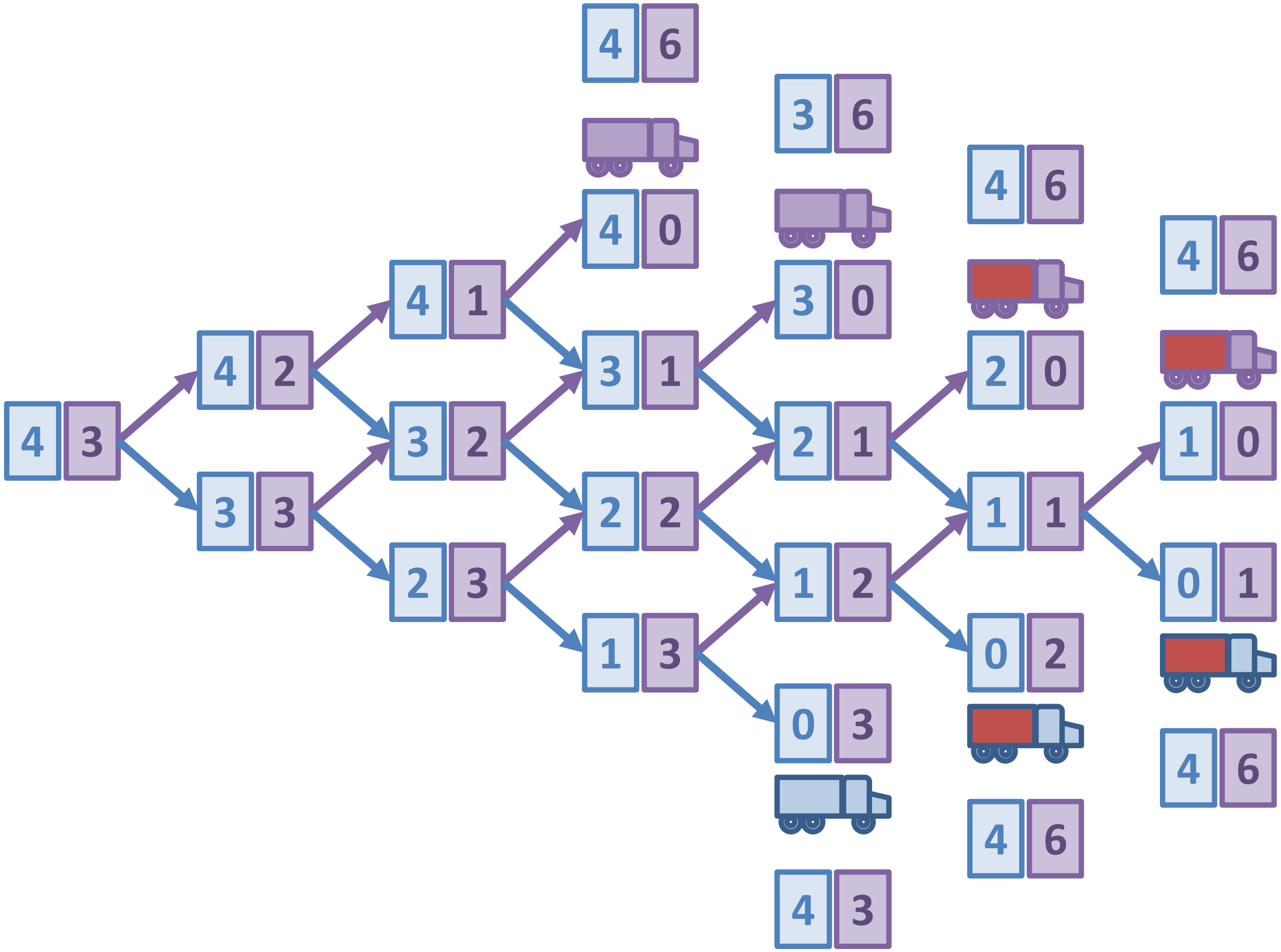
4 4
4 3
3 6



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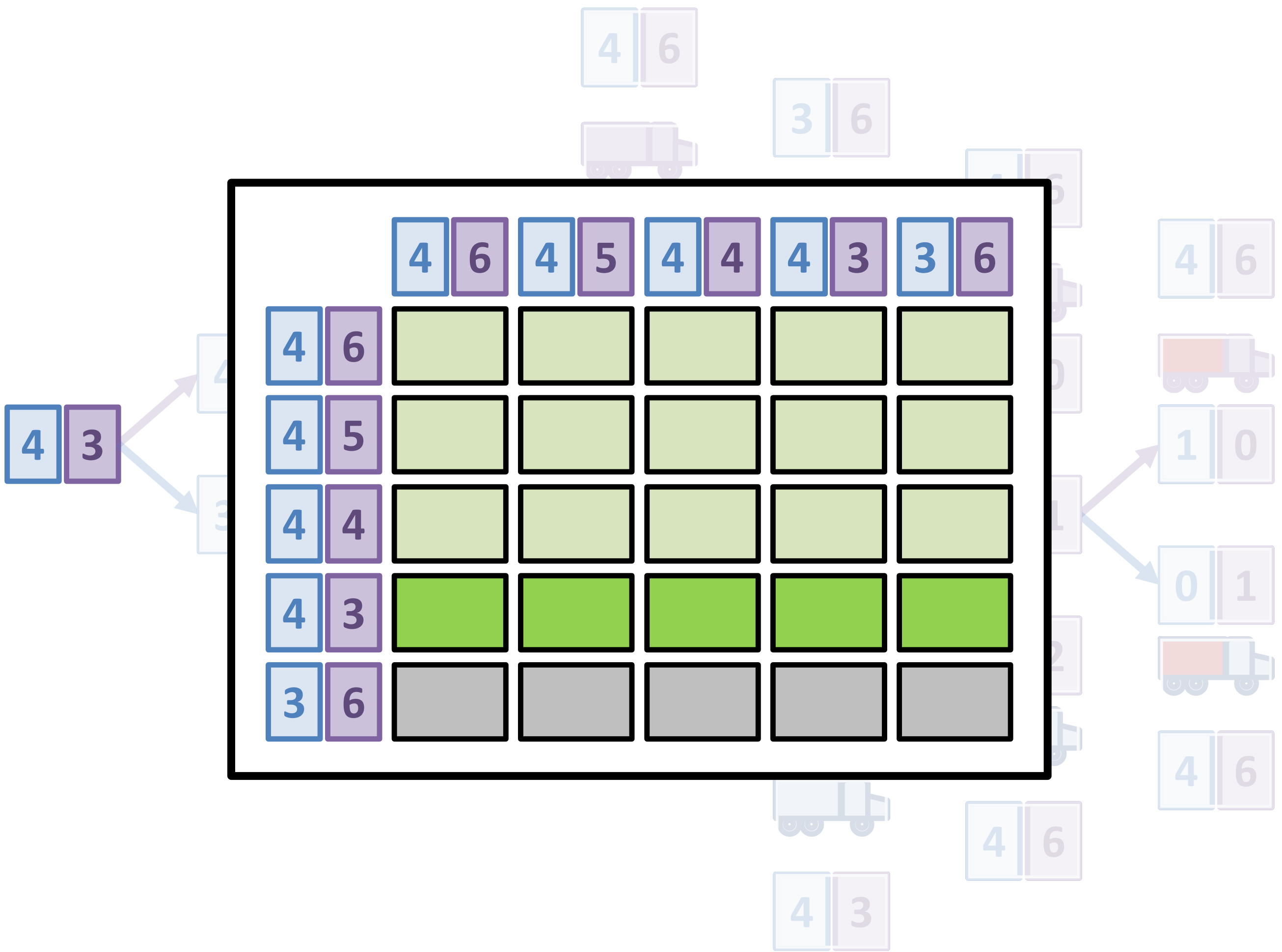
4 6



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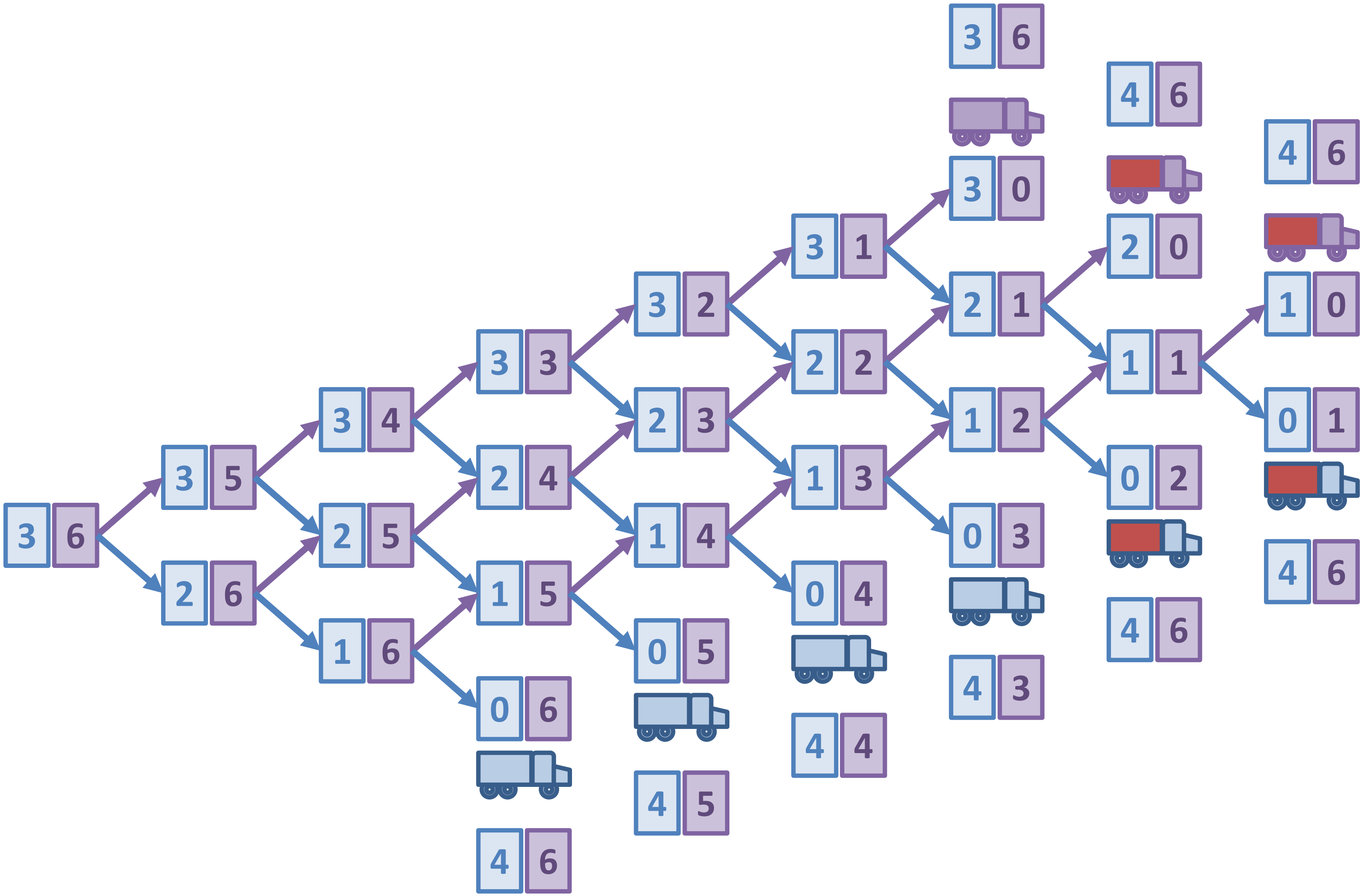
3 6


3 6
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2 4
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4 5
3 1
2 2
1 3
0 4
4 4
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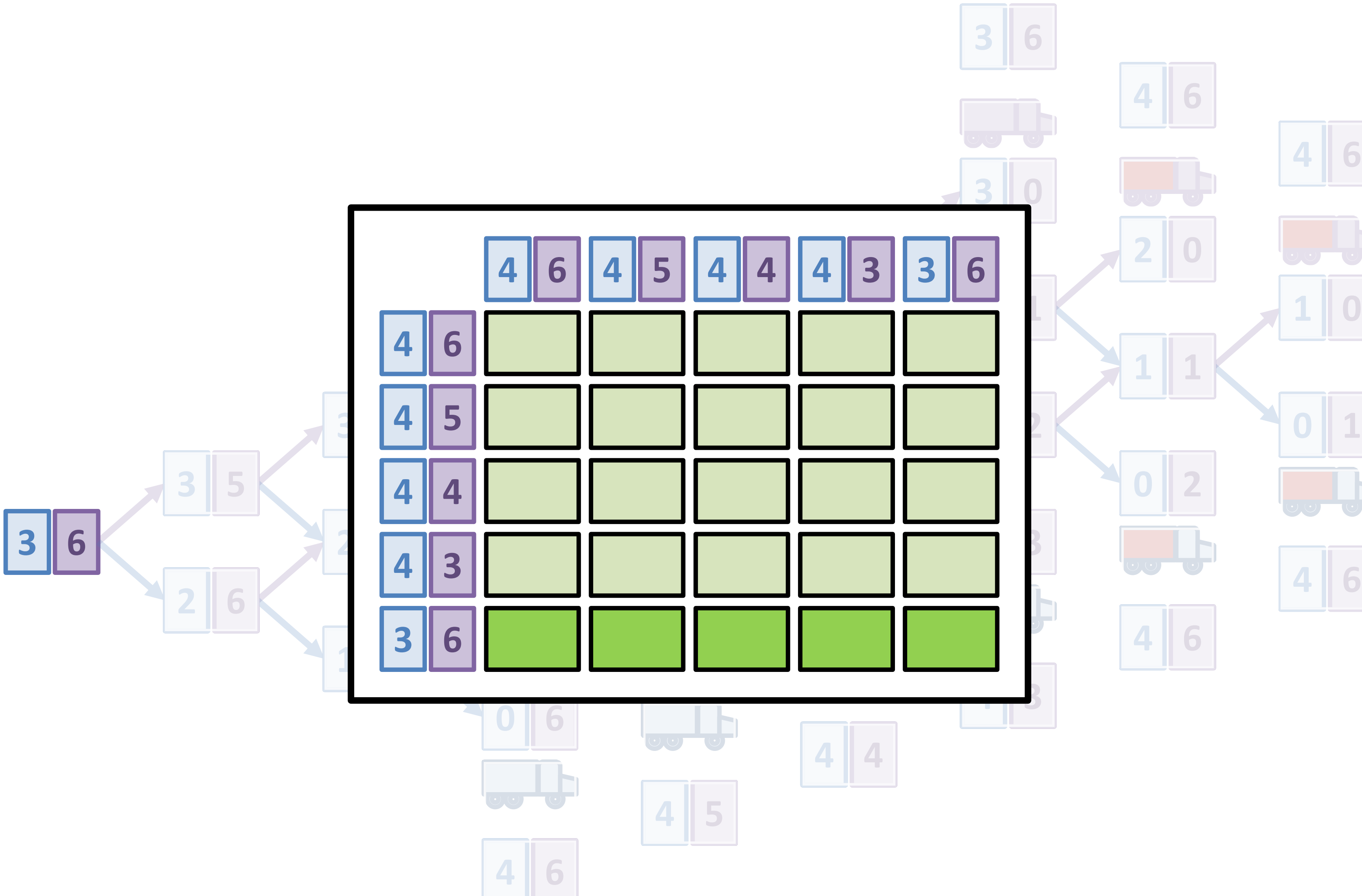
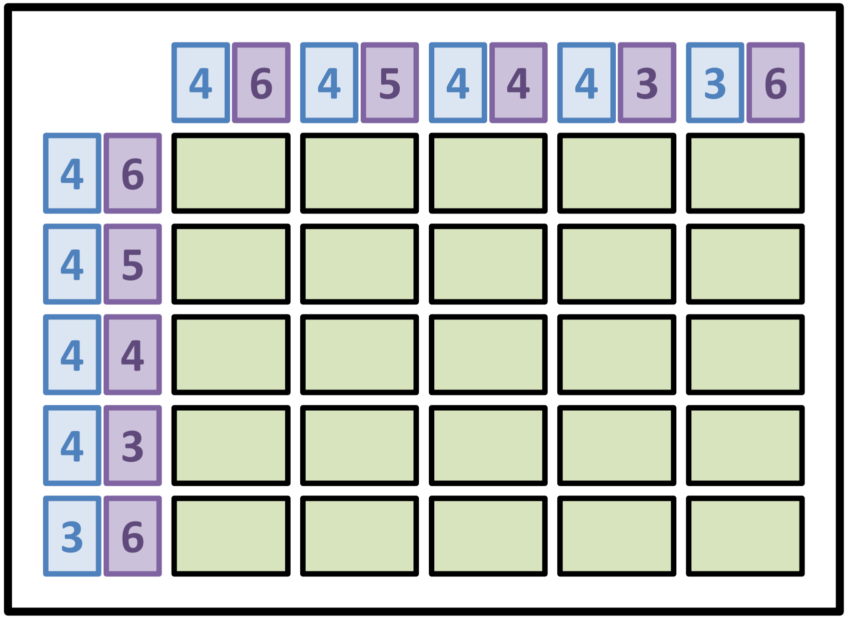
4 6
4 6
1 0
0 1
4 6



We have a Markov chain that holds the probabilities to move from one initial state towards another

	4 6	4 5	4 4	4 3	3 6
4 6					
4 5					
4 4					
4 3					
3 6					

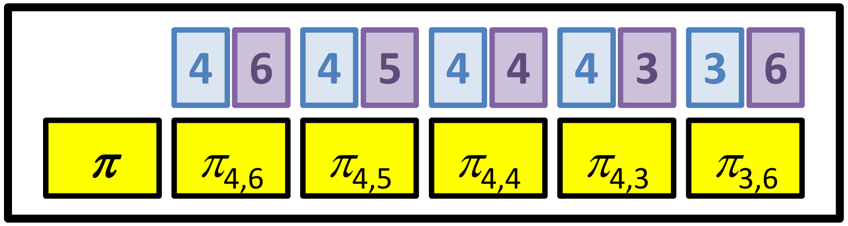
We have a Markov chain that holds the probabilities to move from one initial state towards another



From this Markov chain, we can obtain the steady-state probabilities to visit one of the initial states!

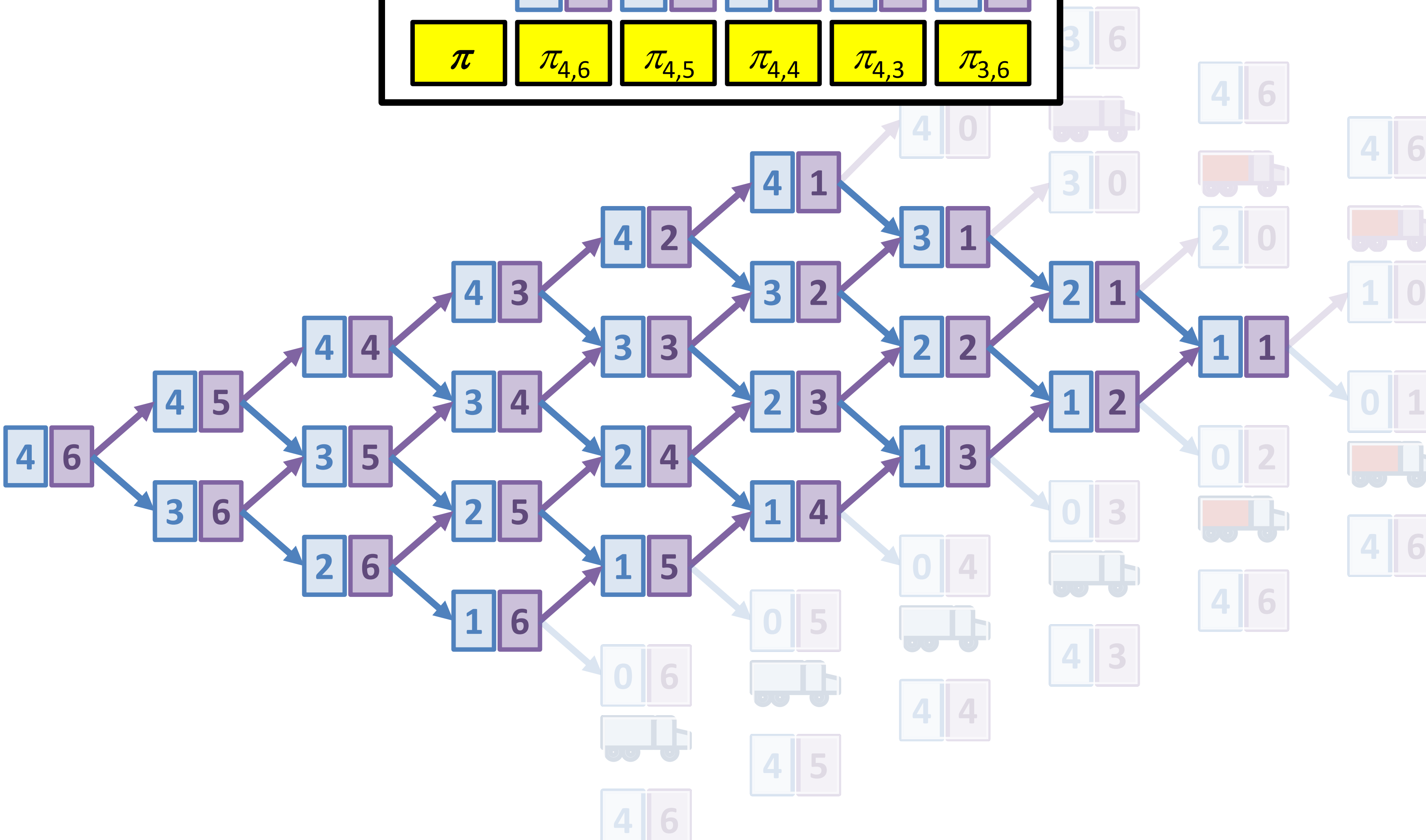
	4 6	4 5	4 4	4 3	3 6
π	$\pi_{4,6}$	$\pi_{4,5}$	$\pi_{4,4}$	$\pi_{4,3}$	$\pi_{3,6}$

We can use these steady-state probabilities to weigh the probability to visit a regular state when departing from a given initial state



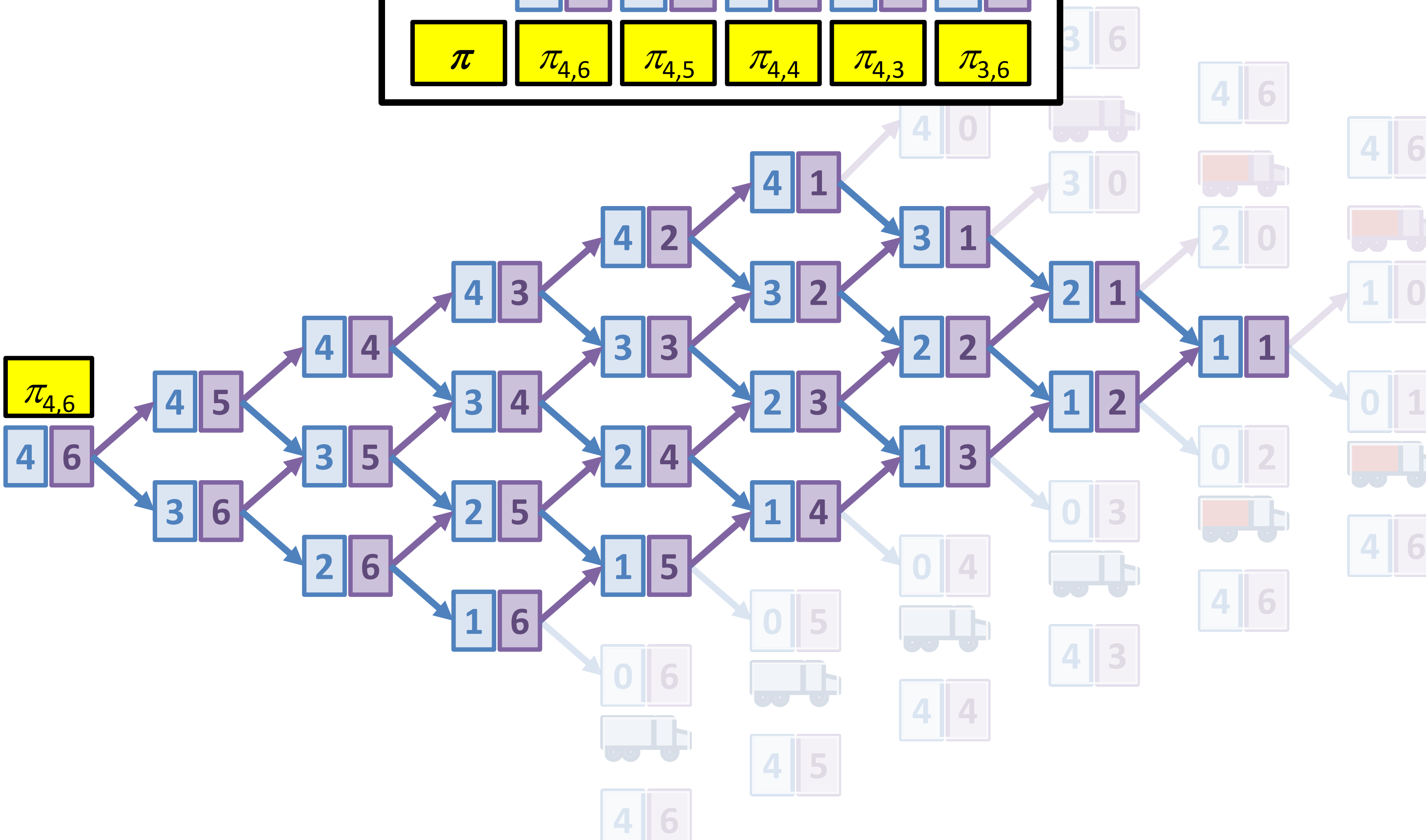
We can use these steady-state probabilities to weigh the probability to visit a regular state when departing from a given initial state

	4 6	4 5	4 4	4 3	3 6
π	$\pi_{4,6}$	$\pi_{4,5}$	$\pi_{4,4}$	$\pi_{4,3}$	$\pi_{3,6}$



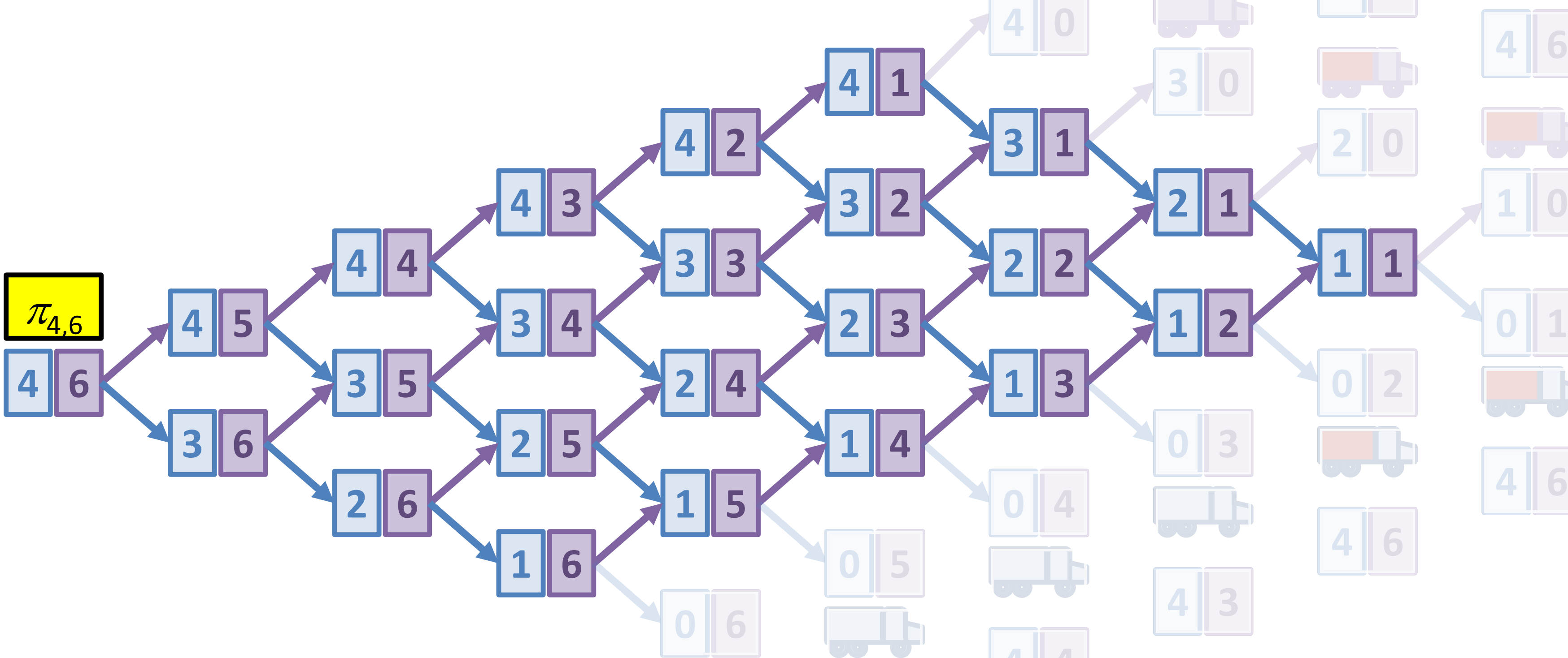
We can use these steady-state probabilities to weigh the probability to visit a regular state when departing from a given initial state

	4 6	4 5	4 4	4 3	3 6
π	$\pi_{4,6}$	$\pi_{4,5}$	$\pi_{4,4}$	$\pi_{4,3}$	$\pi_{3,6}$



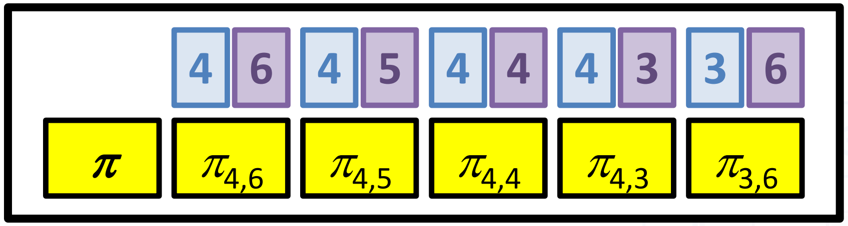
We can use these steady-state probabilities to weigh the probability to visit a regular state when departing from a given initial state

	4 6	4 5	4 4	4 3	3 6
π	$\pi_{4,6}$	$\pi_{4,5}$	$\pi_{4,4}$	$\pi_{4,3}$	$\pi_{3,6}$



Recall that the visit probabilities of the regular states can easily be obtained using the binomial distribution

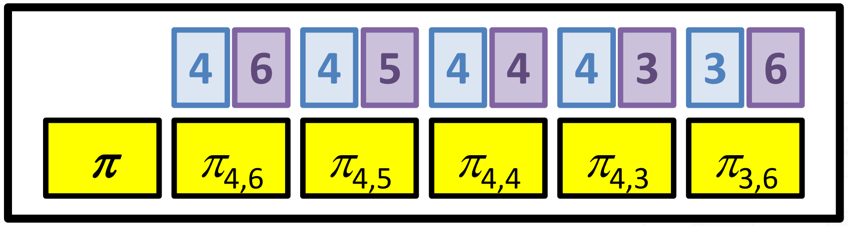
We can use these steady-state probabilities to weigh the probability to visit a regular state when departing from a given initial state



We obtain the steady-state probabilities to visit any of the regular states as the weighted sum of probabilities to visit the regular states when departing from a given initial state

Recall that the visit probabilities of the regular states can easily be obtained using the binomial distribution

We can use these steady-state probabilities to weigh the probability to visit a regular state when departing from a given initial state

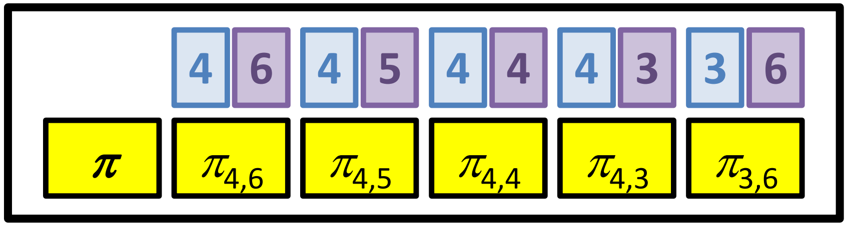


We obtain the steady-state probabilities to visit any of the regular states as the weighted sum of probabilities to visit the regular states when departing from a given initial state

Using the steady-state probabilities to visit the regular states, we can easily calculate the expected inventory at each company

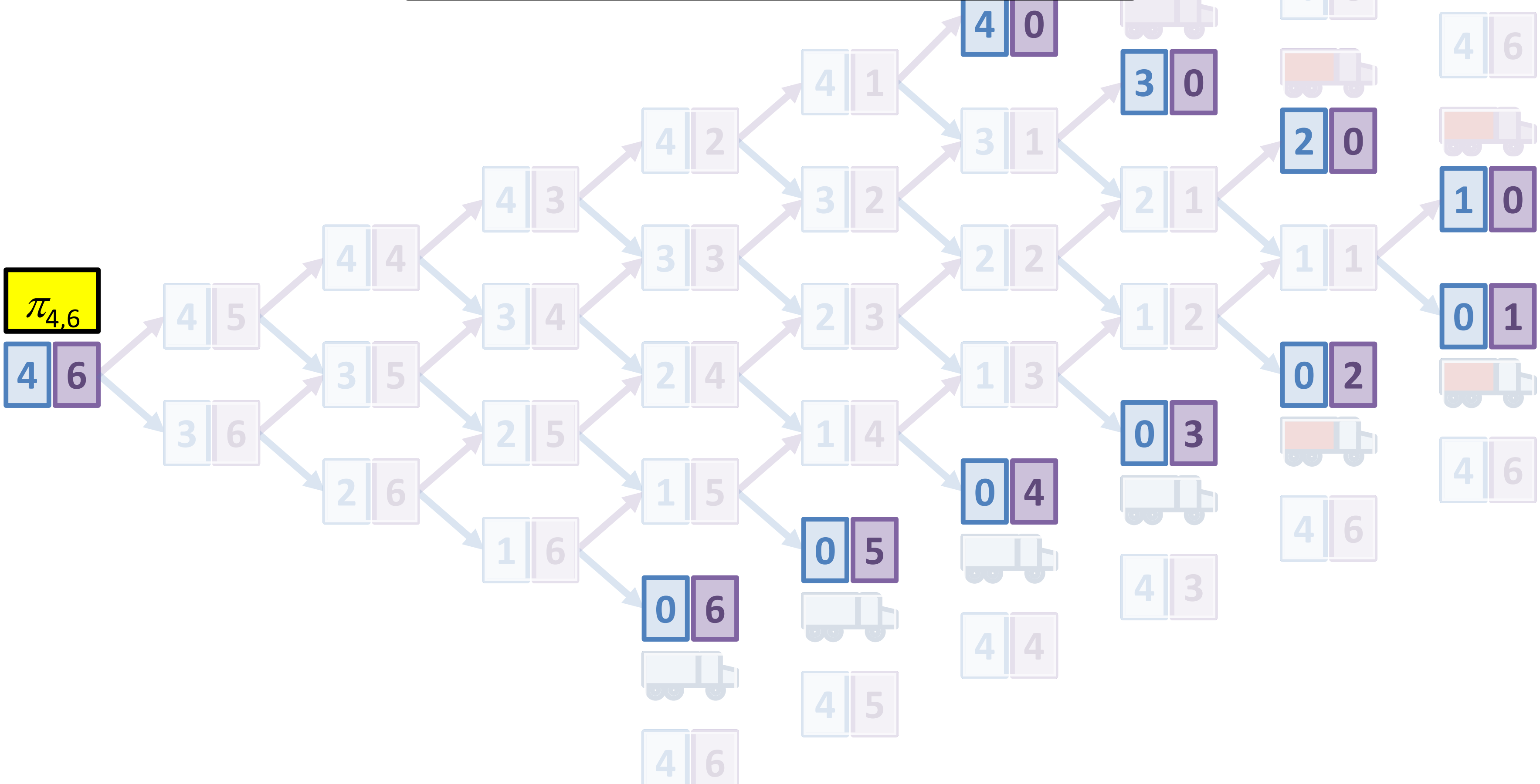
Recall that the visit probabilities of the regular states can easily be obtained using the binomial distribution

We can also use these steady-state probabilities to weigh the probability to visit a final state when departing from an initial state



We can also use these steady-state probabilities to weigh the probability to visit a final state when departing from an initial state

	4 6	4 5	4 4	4 3	3 6
π	$\pi_{4,6}$	$\pi_{4,5}$	$\pi_{4,4}$	$\pi_{4,3}$	$\pi_{3,6}$



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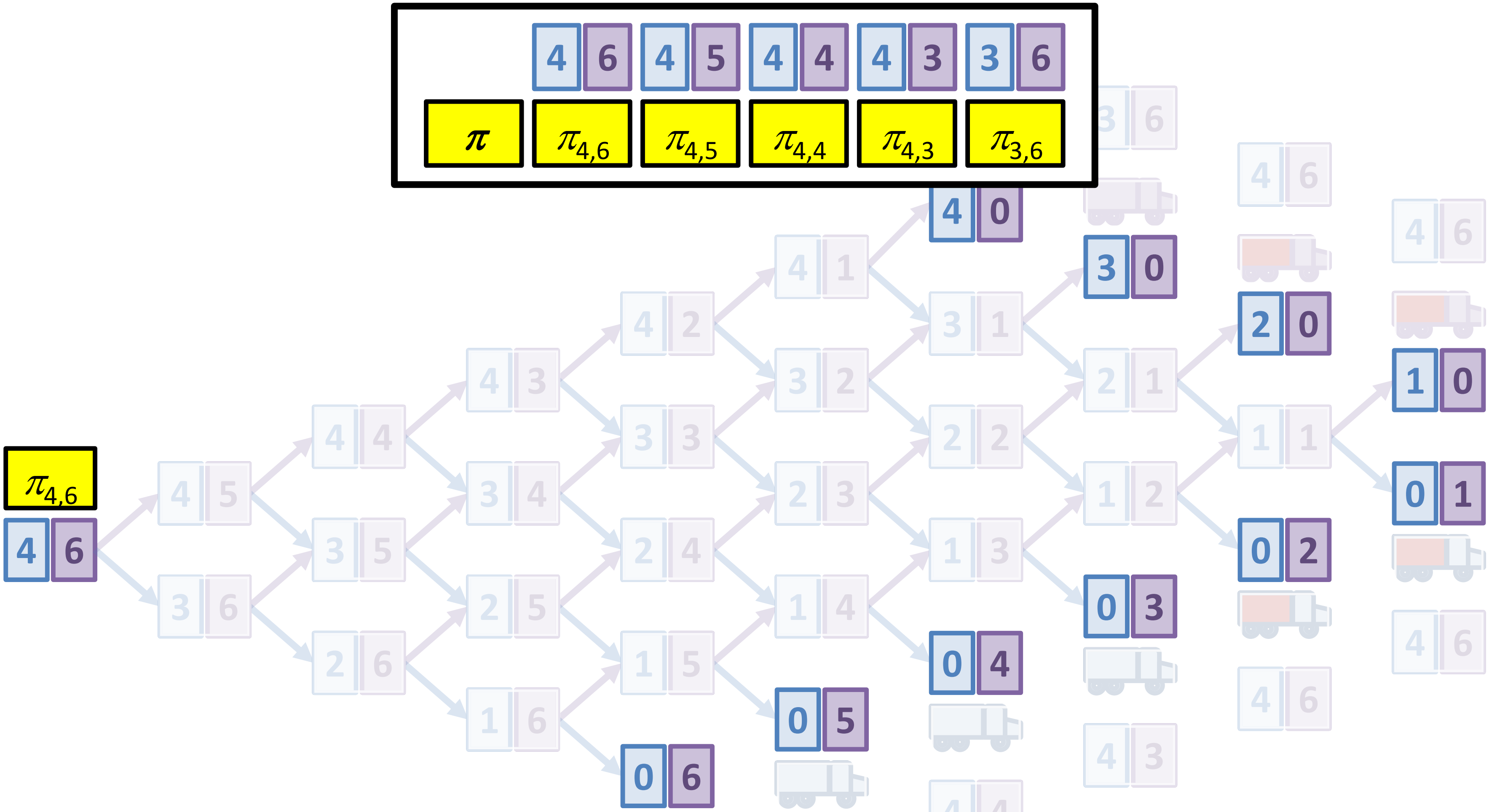

Recall that the visit probabilities of the final states can easily be obtained using the negative binomial distribution

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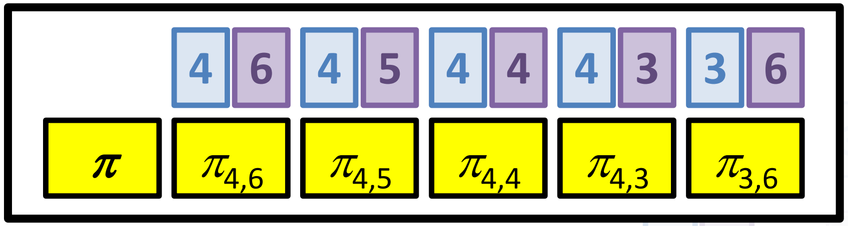
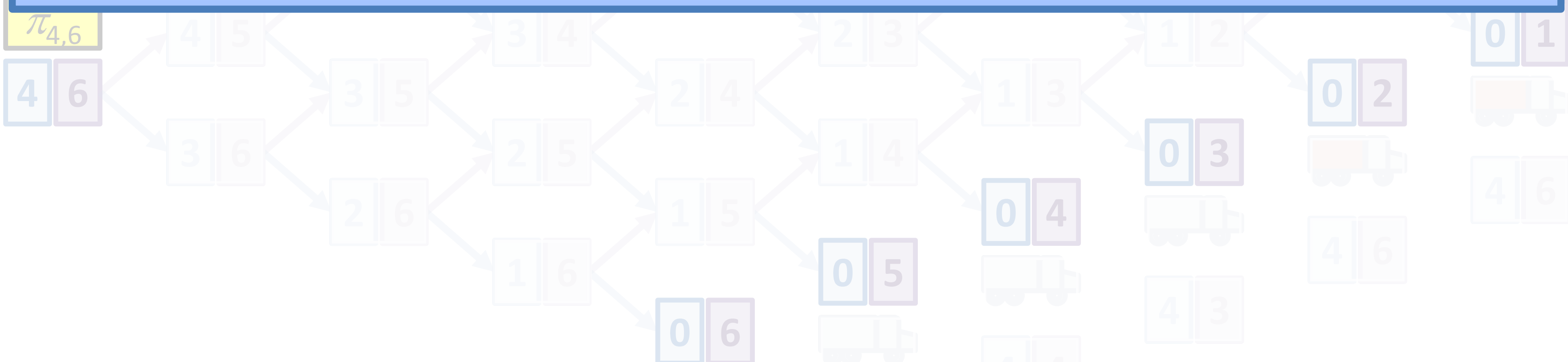


Again, we obtain the steady-state probabilities to visit any of the final states as the weighted sum of probabilities to visit the final states when departing from a given initial state



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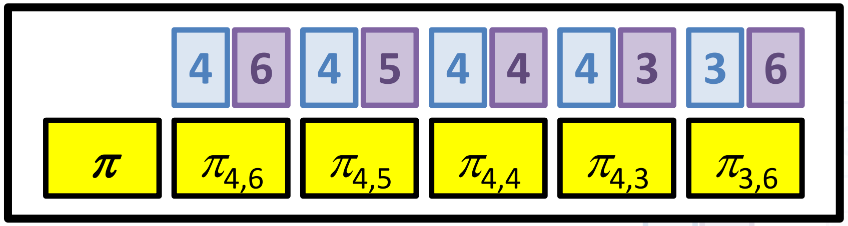

Again, we obtain the steady-state probabilities to visit any of the final states as the weighted sum of probabilities to visit the final states when departing from a given initial state

Given the number of transitions it takes to move from an initial state to a final state, we can calculate the number of times a company places a single/joined order

Recall that the visit probabilities of the final states can easily be obtained using the negative binomial distribution

Numerical Example: Conclusions

- If we use a regular Markov chain to model the example:
  - We end up with 24 states
  - We cannot easily calculate the number of orders (joined/single) for each company
- If we use our new approach:
  - We end up with a Markov chain of 5 states
  - We can easily obtain both inventory holding costs and order costs (i.e., the total cost of the coordination)

Agenda

- Horizontal cooperation: what, why, how?
- Examples of horizontal cooperations
- Definitions & assumptions
- Problem setting example
- Costs & performance measures
- Methodology
- Numerical example
- Future research

Future/Current Research

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- Because our model is fast/efficient, we can use it to study the characteristics of the optimal policy in a two-company horizontal cooperation
- Lastly, we also relax the assumptions:
  - Non-zero & non-exponential lead times
  - Non-exponential customer interarrival times
  - (S,c,Q) order policy
  - Truck capacity constraints

?