Project Scheduling for Maximum NPV with Variable Activity Durations and Uncertain Activity Outcomes

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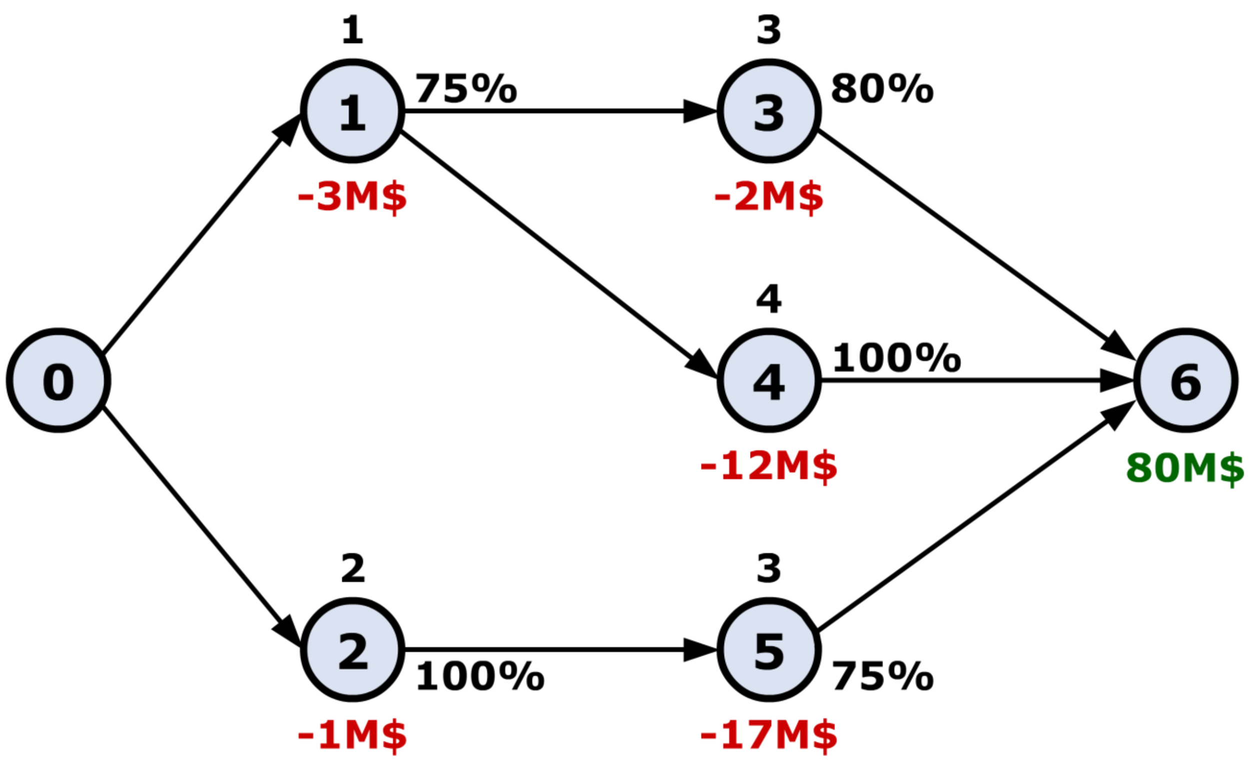
Introduction:
Activity failure

- Common to many R&D-projects (especially NPD), but also occurs in other sectors: pharmaceuticals, chemicals, construction industry, software development, innovation, ...
- Individual activity failure results in overall project failure
  => project pay-off is not obtained
  - FDA review
  - toxicology tests
  - undesirable side effects
  - building permit
  - loan requests
  - market potential
  - patent infringement
  - …

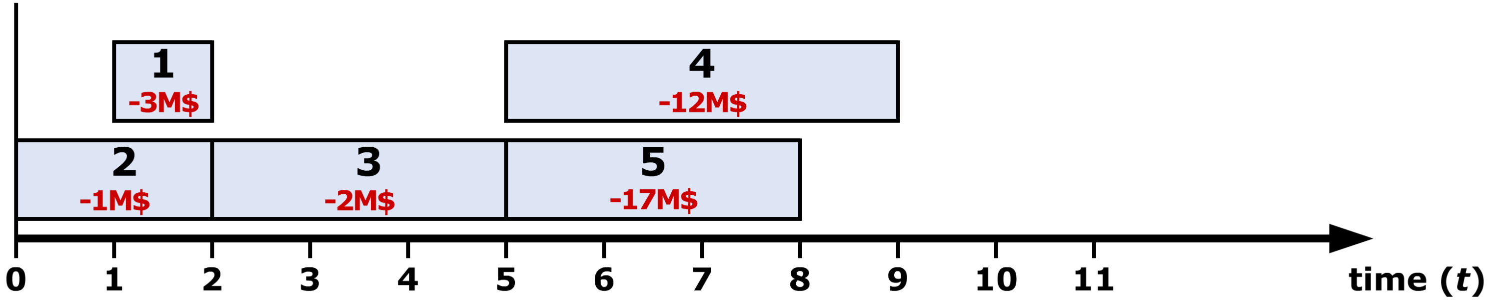
Problem Description:
Example



Problem description:

Example: deterministic durations

Feasible schedule



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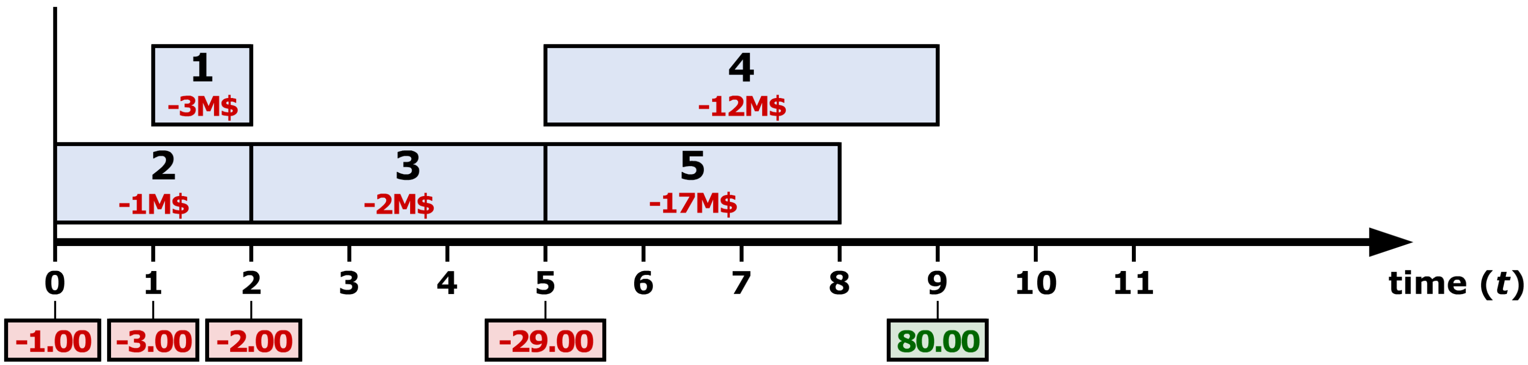

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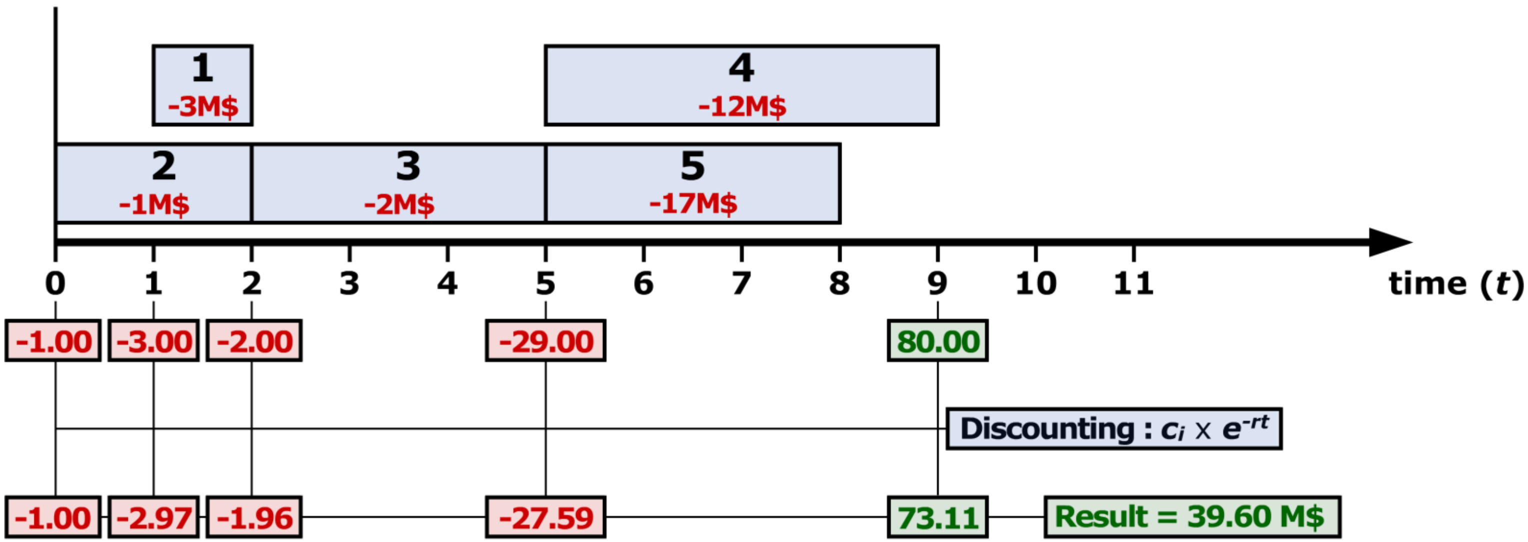

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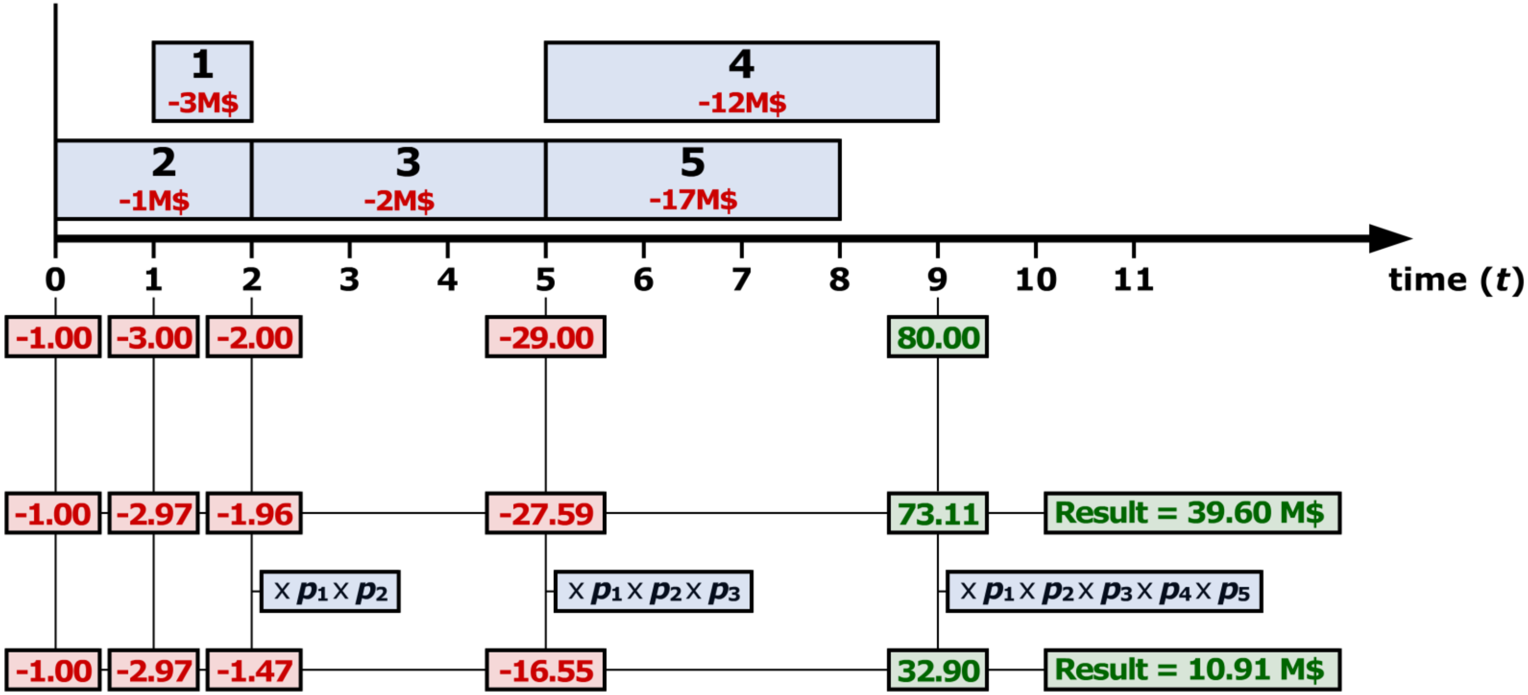
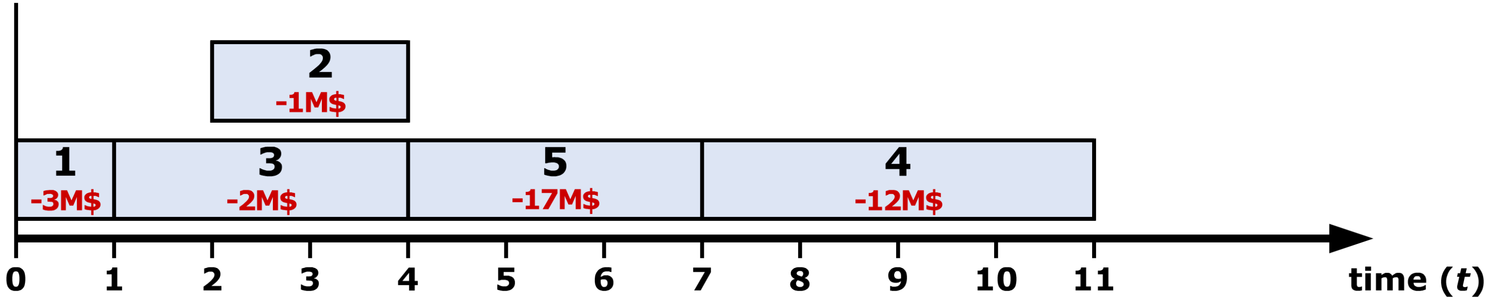


Problem description:
Example: deterministic durations

Optimal schedule



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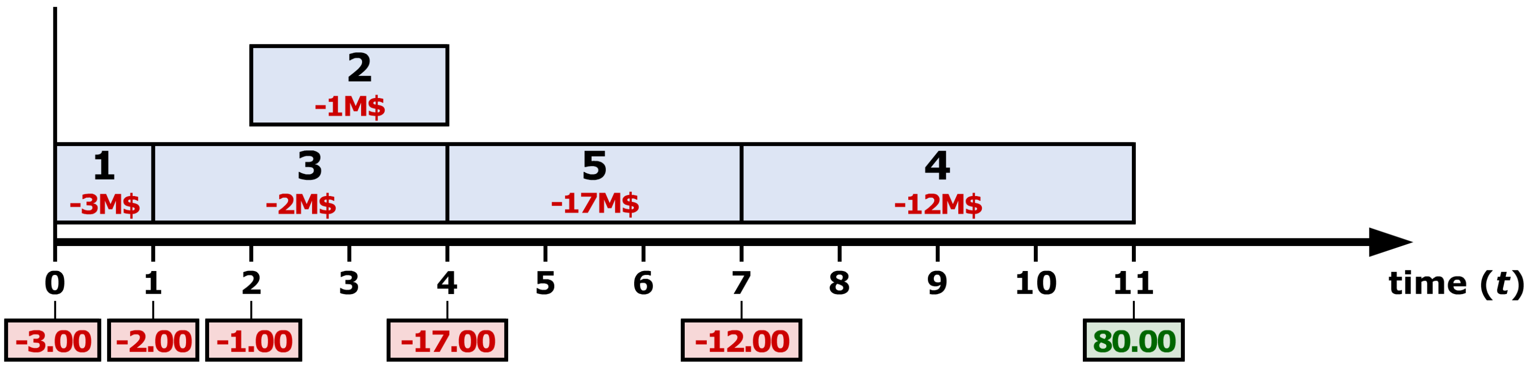

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Example: deterministic durations

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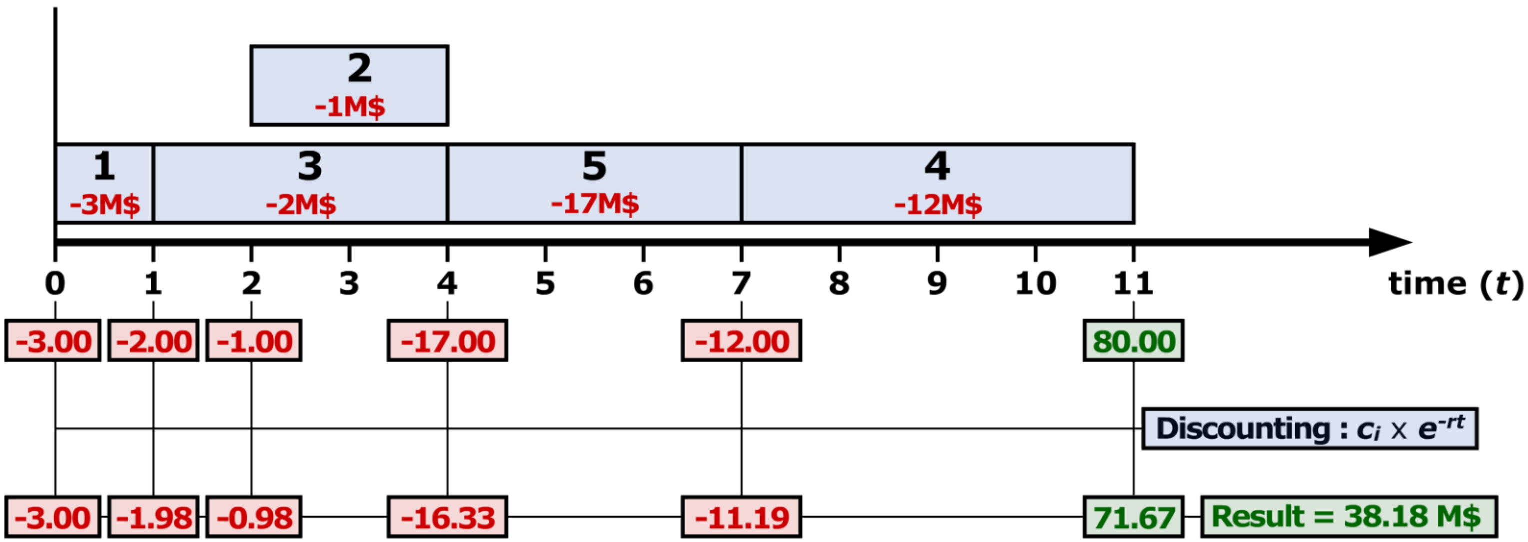

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Example: deterministic durations

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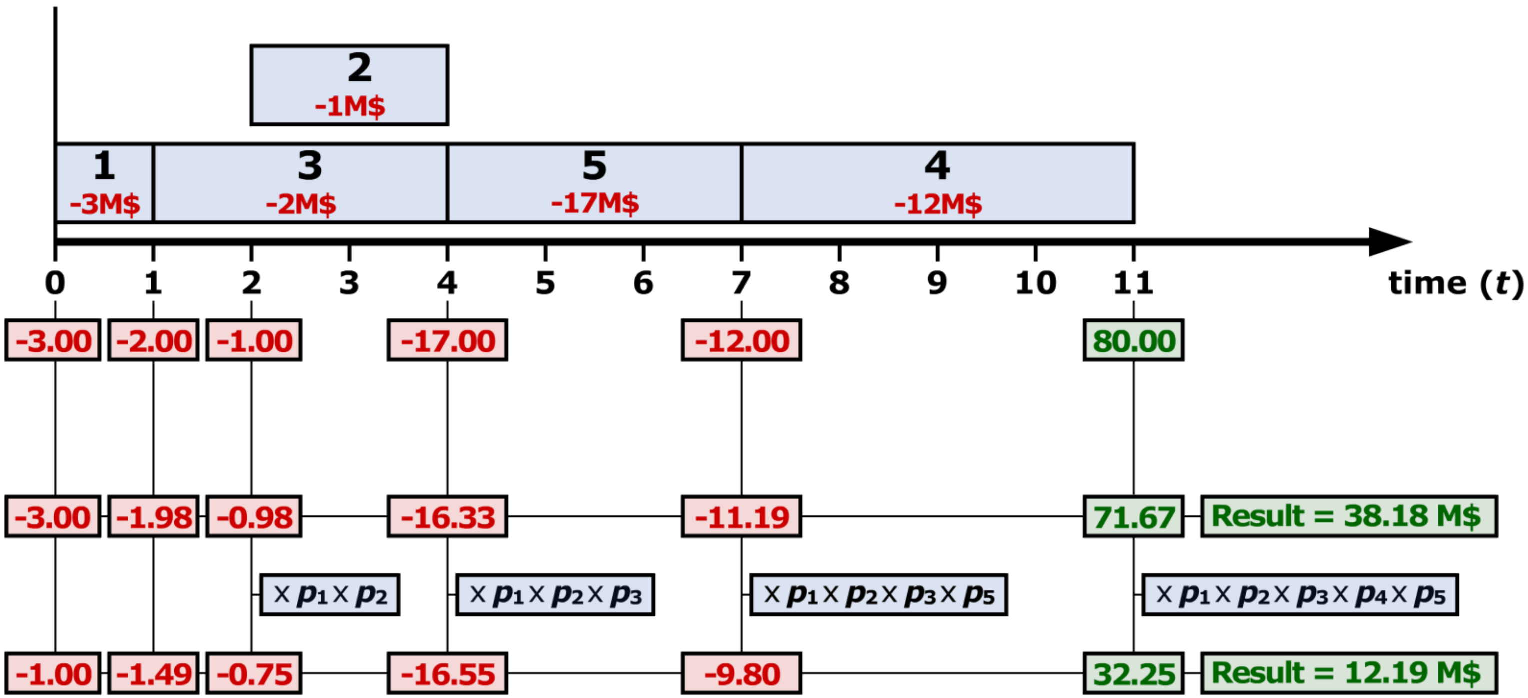


Problem Description:
Definitions

- Stochastic activity durations (exponentially distributed) => use of a Continuous Time Markov Decision Chain
- Expected-NPV-objective: incurred cash flow c_i at the start of activity i
- Optimization over the set of policies that start activities at the end of other activities
- Number of activities n
- Mean duration d_i of activity i
- Activity i has probability of technical success p_i
- Discount rate r
- No renewable resource constraints

Model Description:

Stochastic durations – Continuous time Markov decision chain

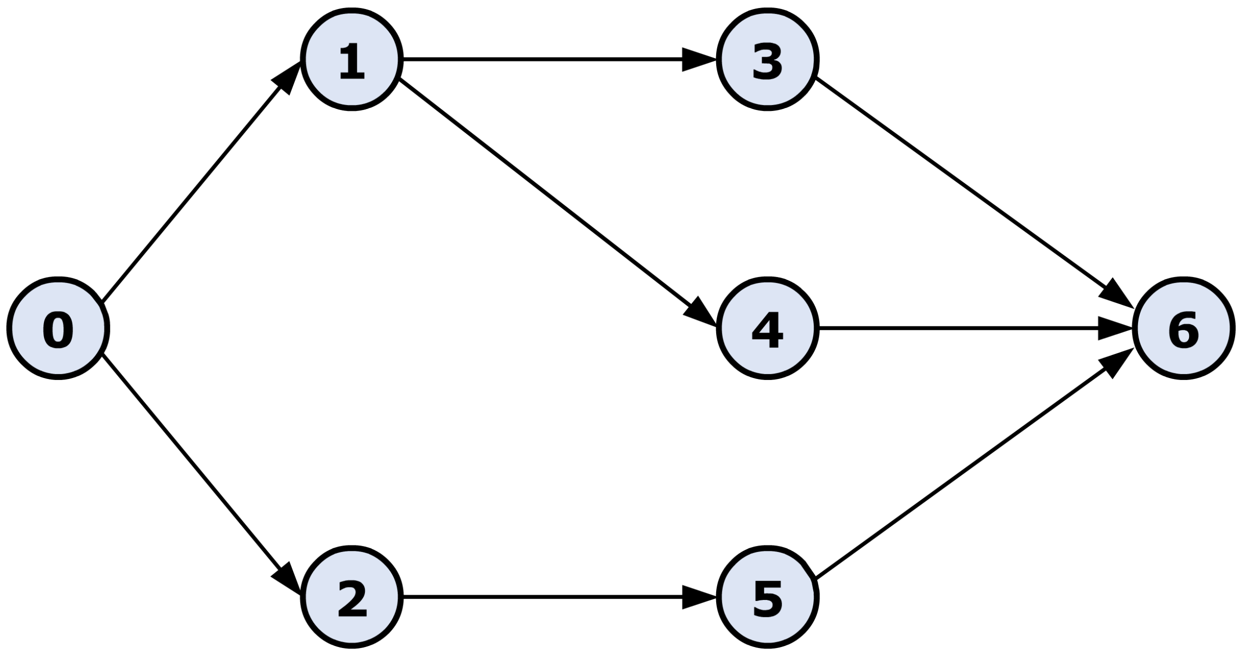
- Preliminary concepts:
  - Status of activity *i* at time *t*:
    - Not started $\Omega_i(t)=0$
    - Started/in progress $\Omega_i(t)=1$
    - Finished $\Omega_i(t)=2$
  - $\Omega(t)=(\Omega_0(t), \Omega_1(t), \ldots, \Omega_n(t))$ defines the state of the system

Size of statespace $\boldsymbol{Q}$ has upper bound $|\boldsymbol{Q}|=3^n$

Most of these states do not satisfy precedence constraints
=> a strict and clear definition of the statespace is essential
=> use of UDC-concept to define the statespace

Model Description:

UDC: max set of activities that can be executed in parallel



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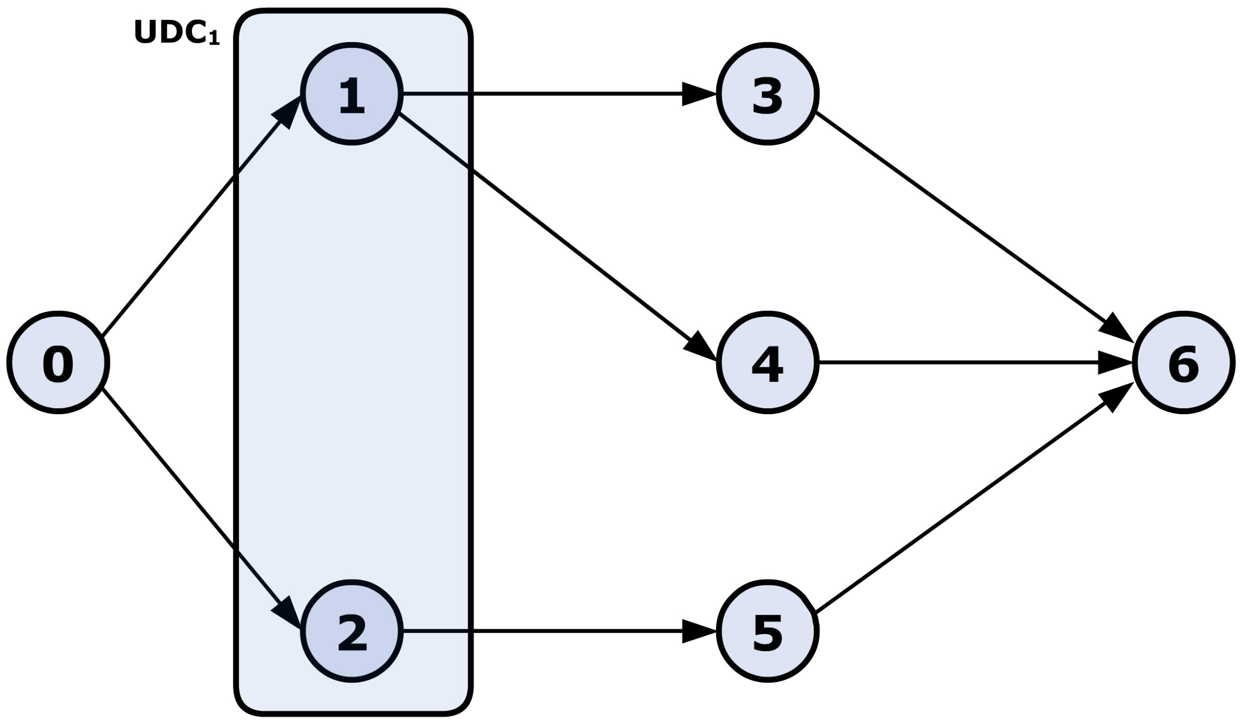

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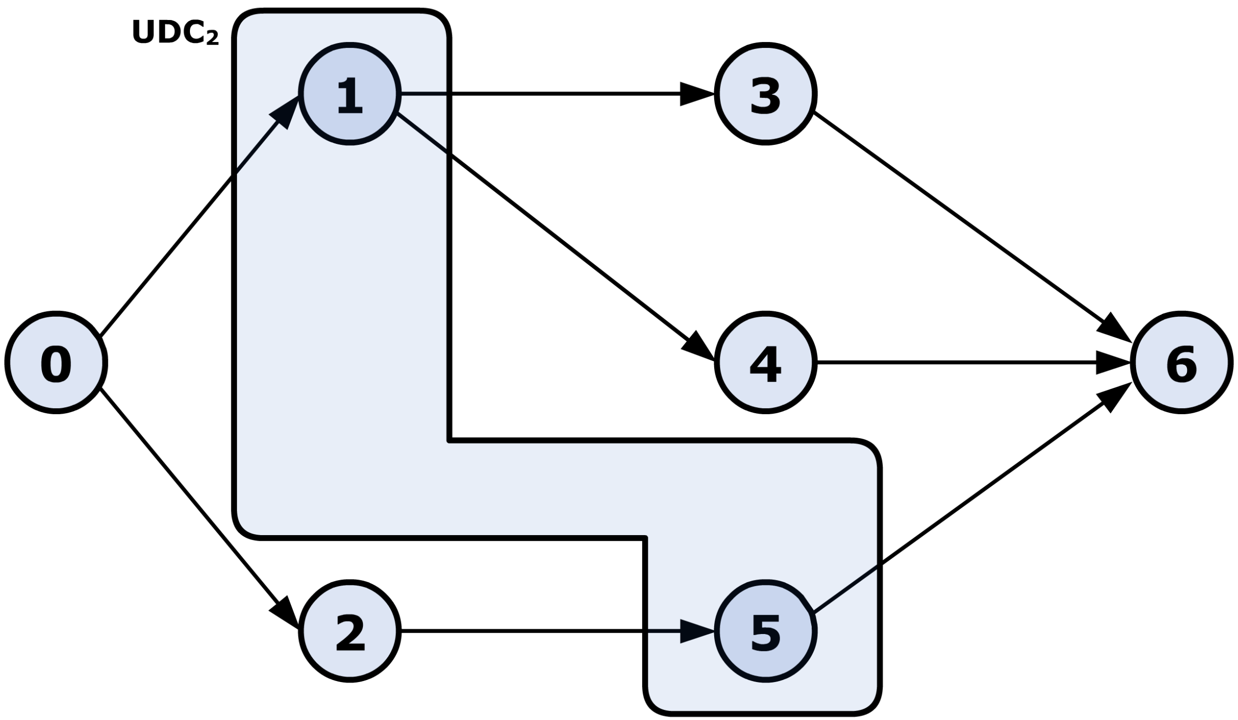

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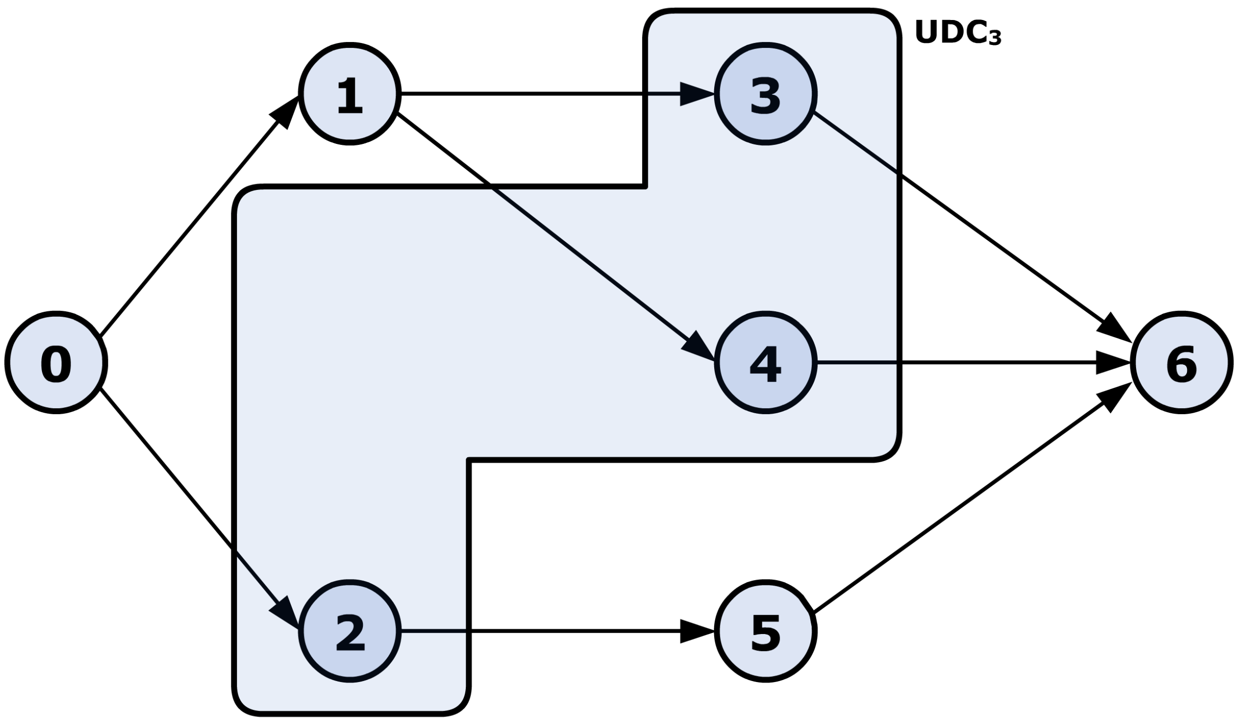

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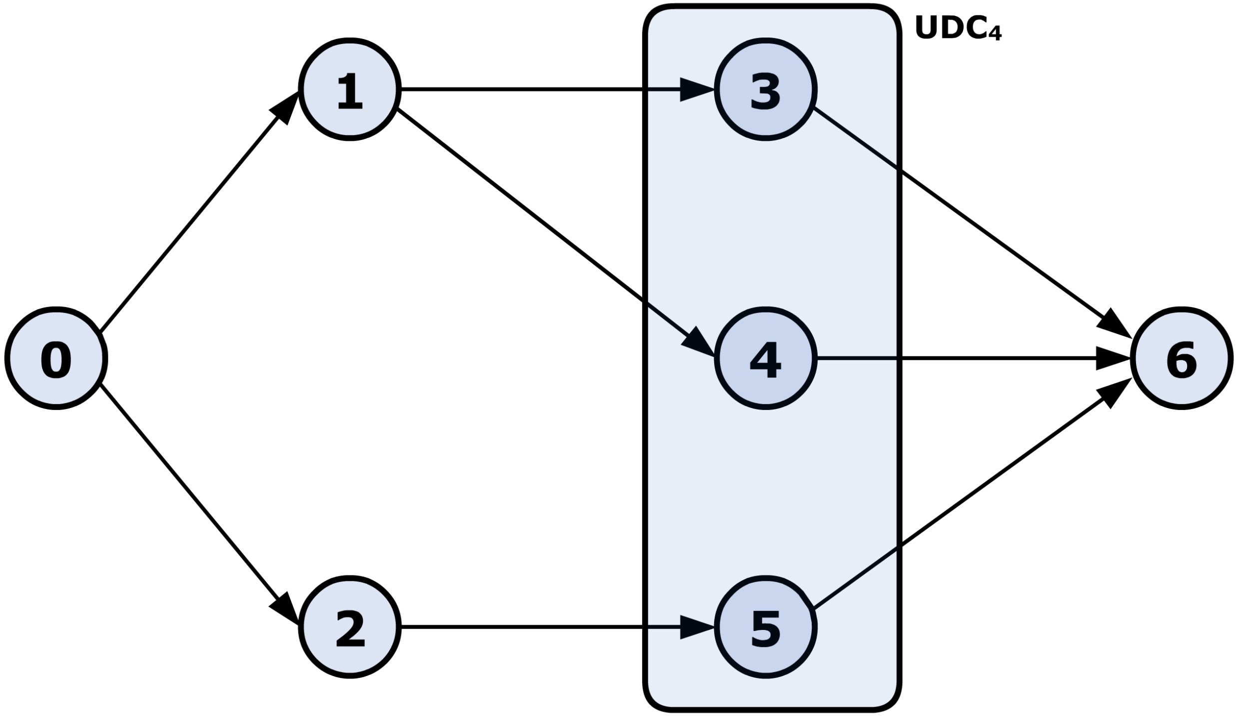

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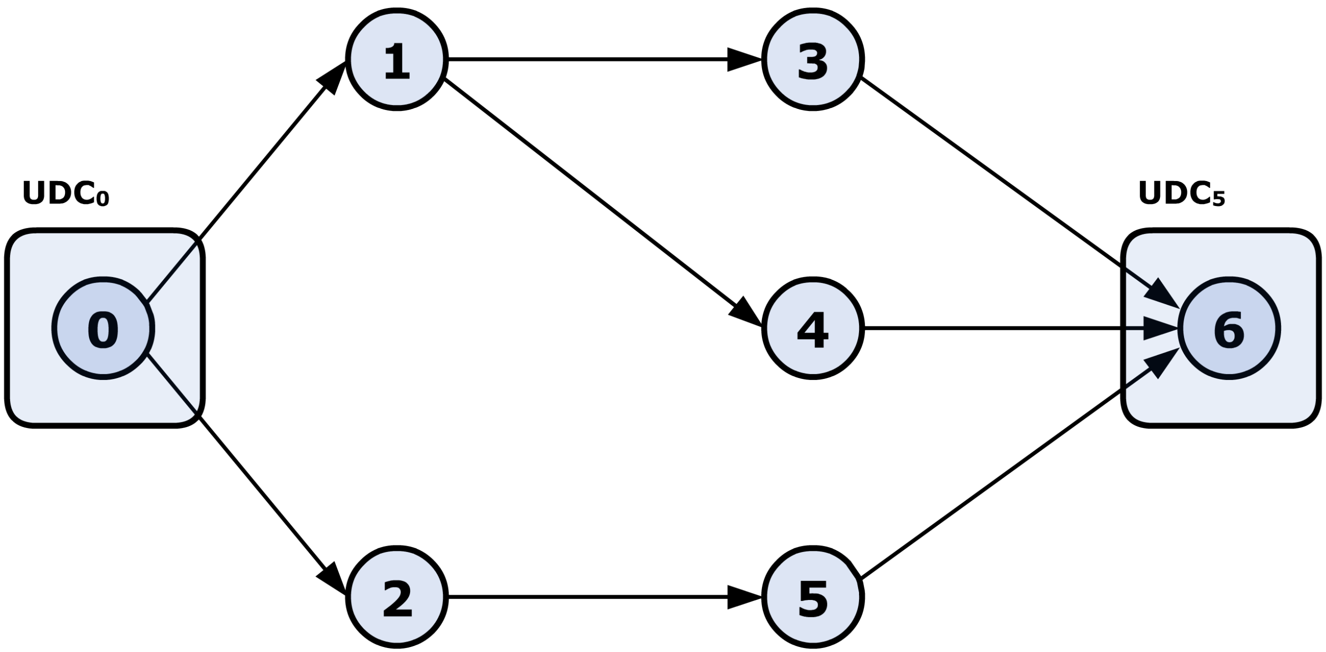
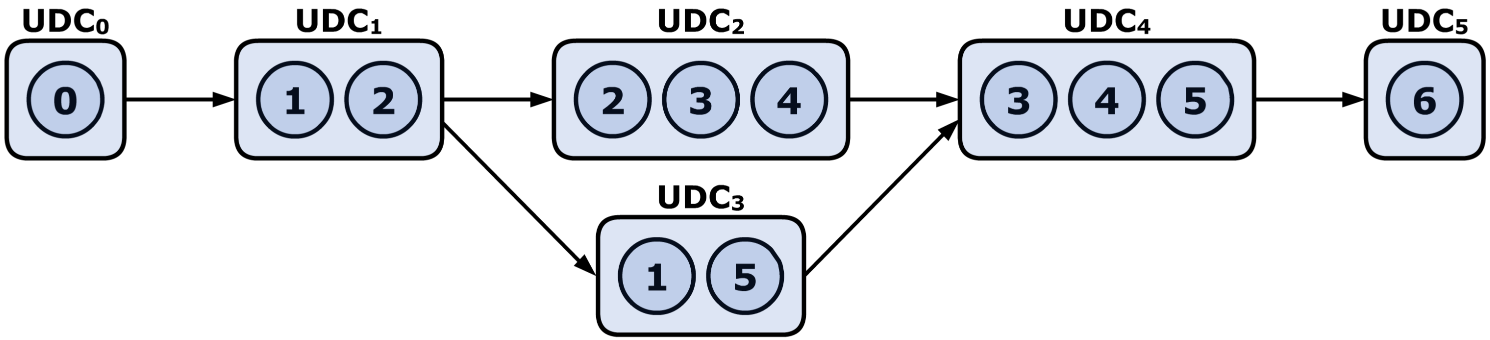
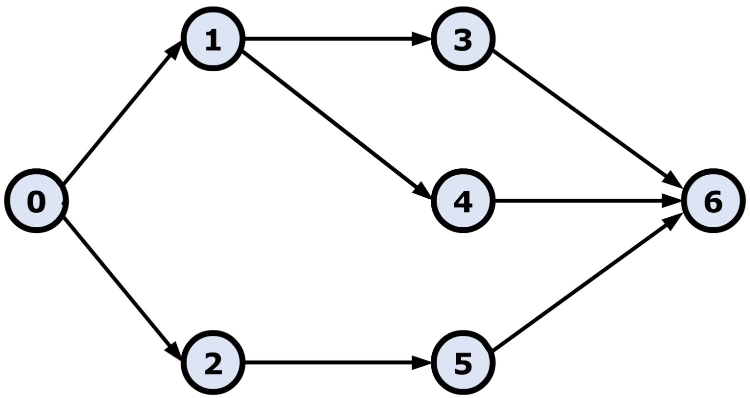


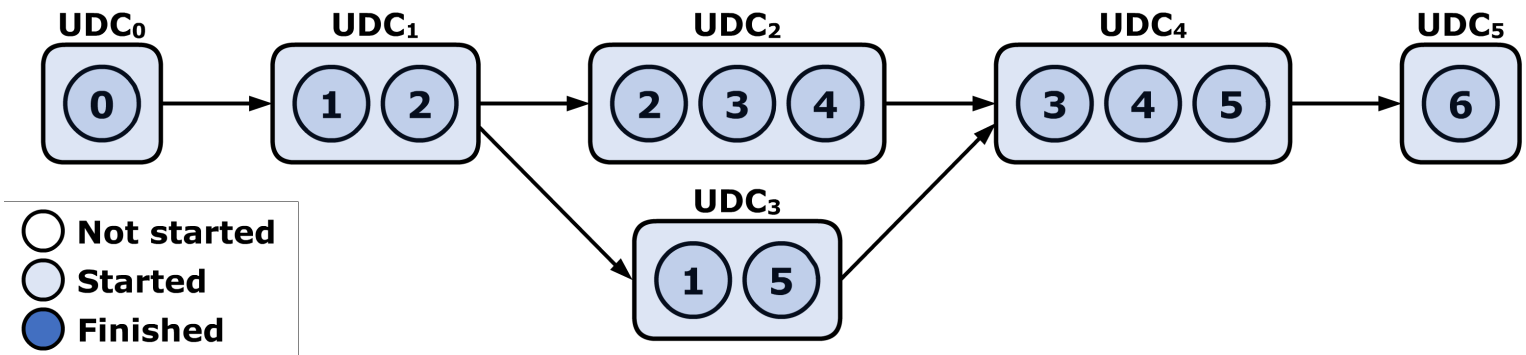
Model Description:
Network of UDCs

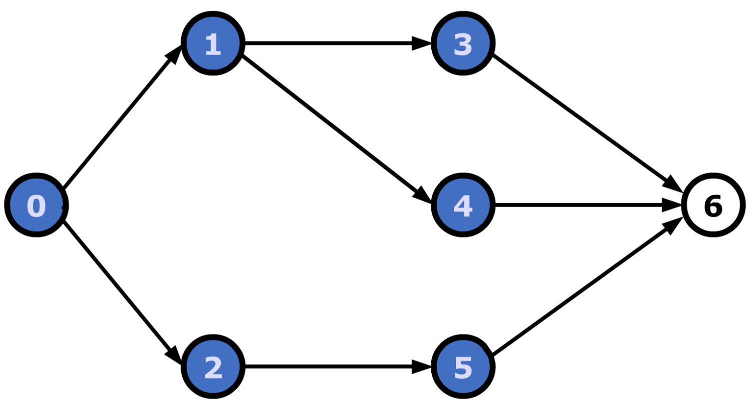




Model Description:

Illustration of statespace and backward SDP-recursion





States assigned to UDC:

(2,2,2,2,2,2,0)

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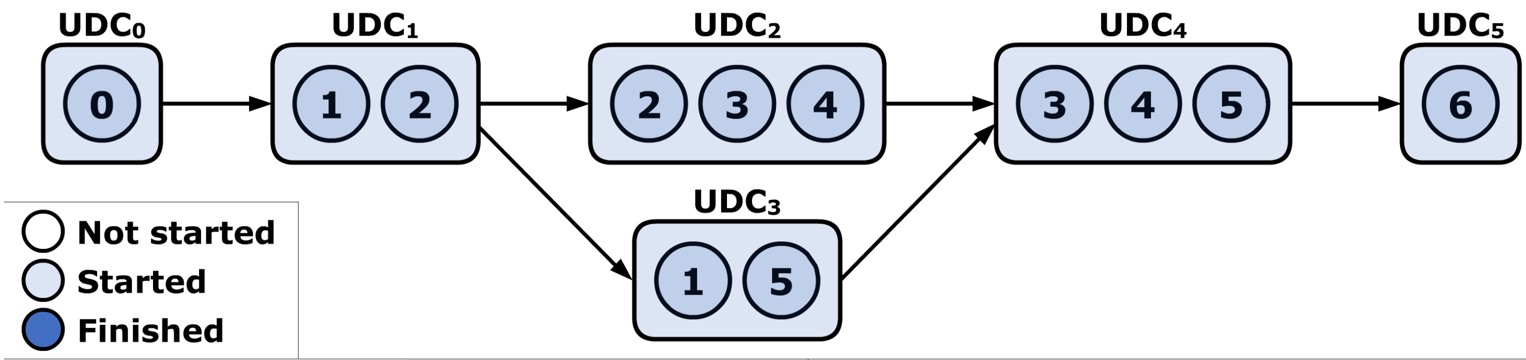

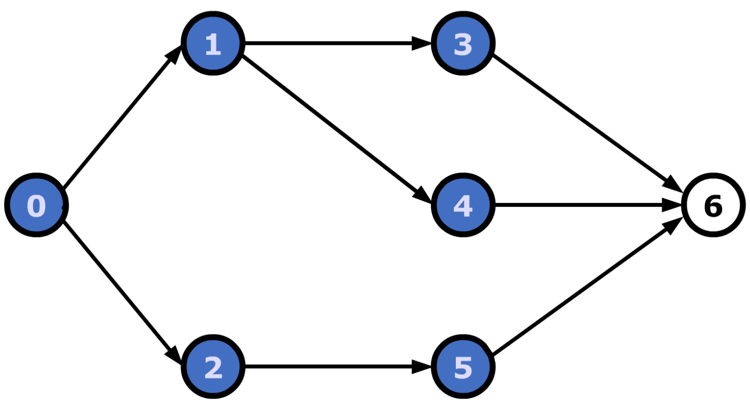

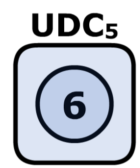

States assigned to UDC:

(2,2,2,2,2,2,0) -> 80M$

Model Description:

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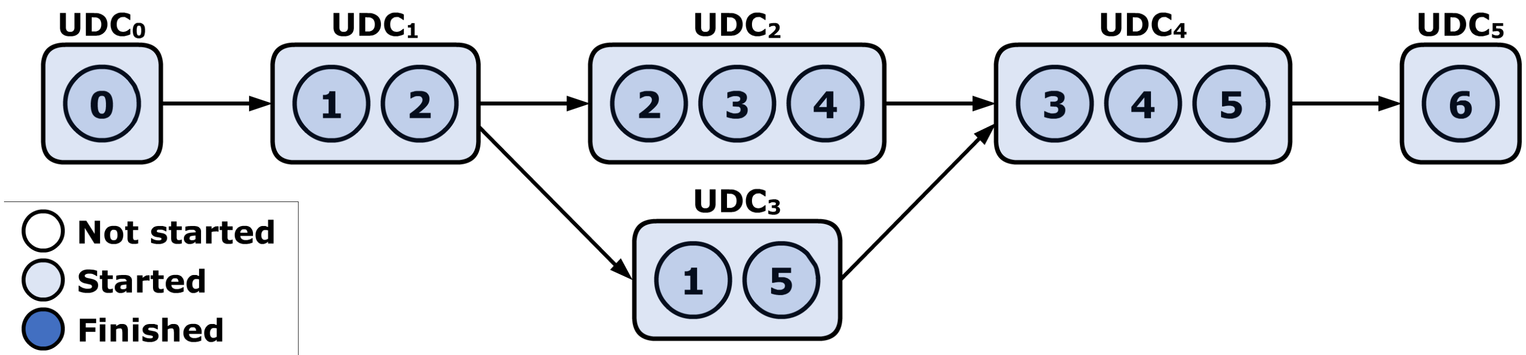

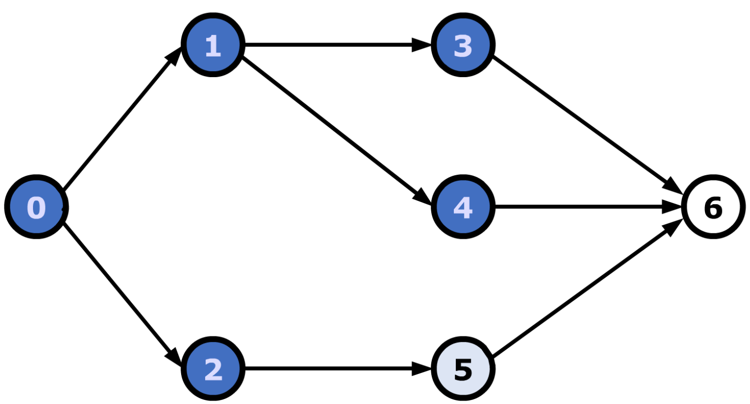

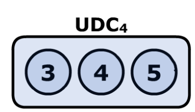

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Model Description:

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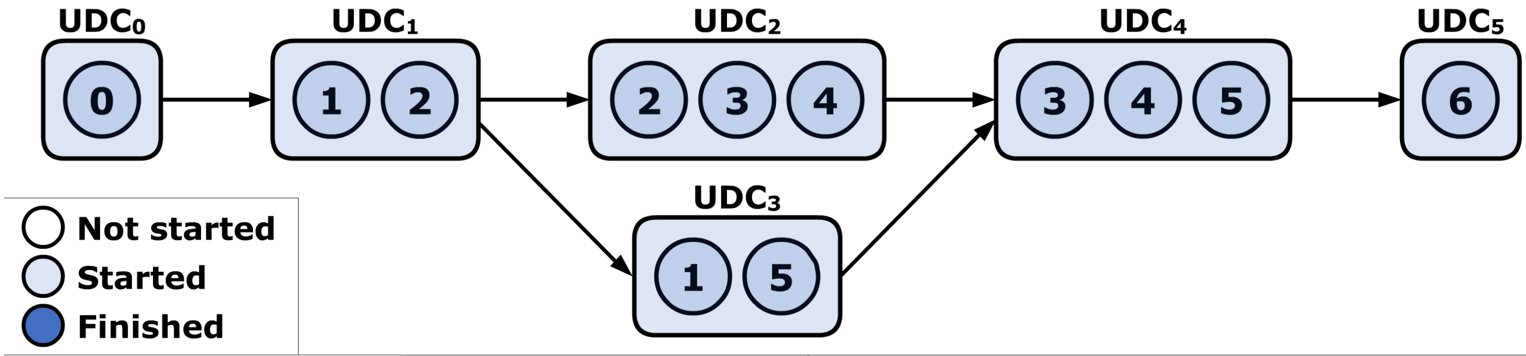

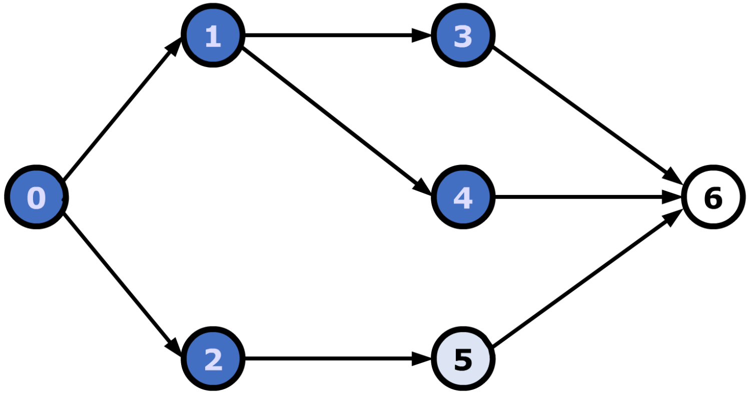

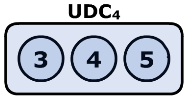

States assigned to UDC:

(2,2,2,2,2,1,0)

└→(2,2,2,2,2,2,0) [80.00M$]

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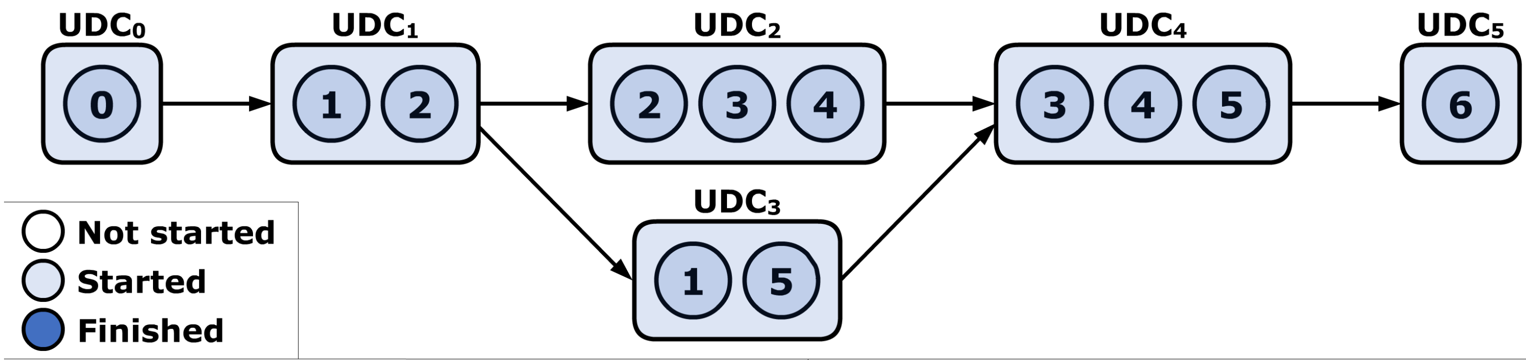

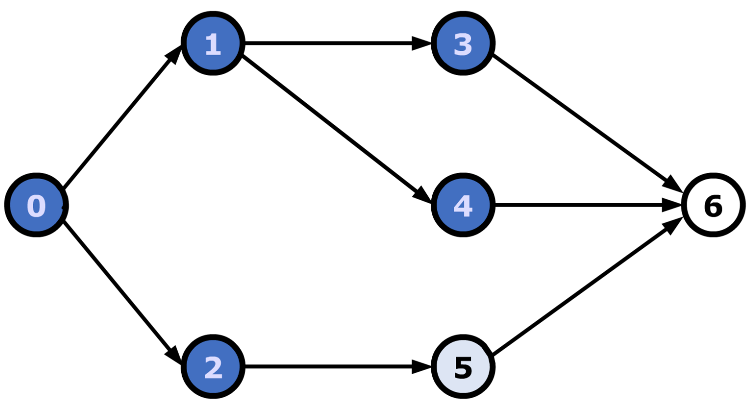

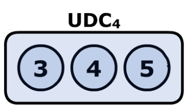

States assigned to UDC:

(2,2,2,2,2,1,0)

└→(2,2,2,2,2,2,0) [80.00M$]

Discount factor : $(1/d_i)\times(r+(1/d_i))^{-1}$

d_5 = 3 => Discount factor = 0.97

Discounted value at state entry = 77.67M$

p_5 = 0.75 => NPV at state entry = 58.25M$

Model Description:

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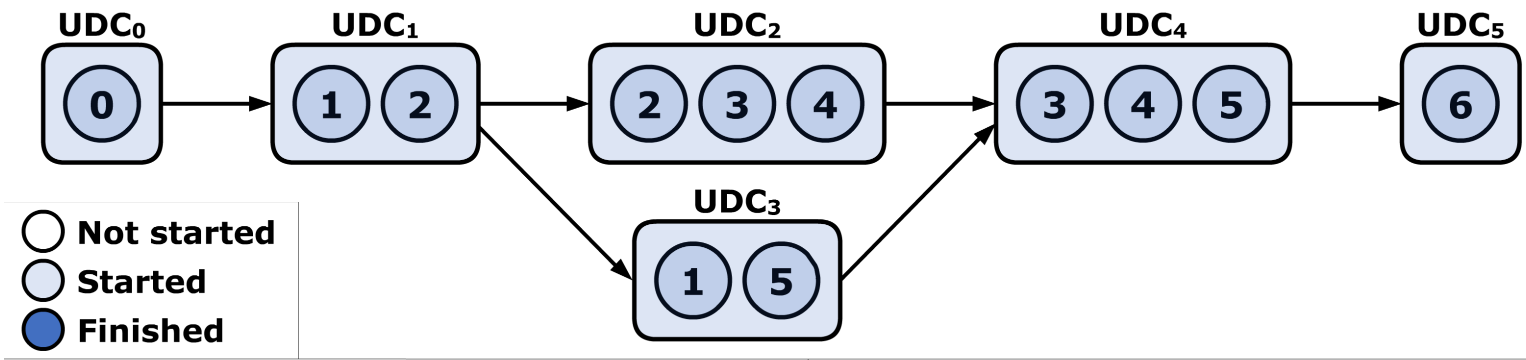

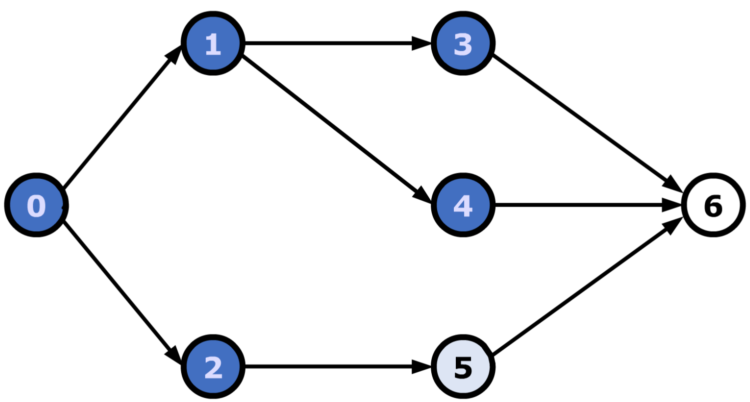

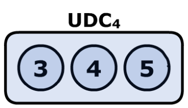

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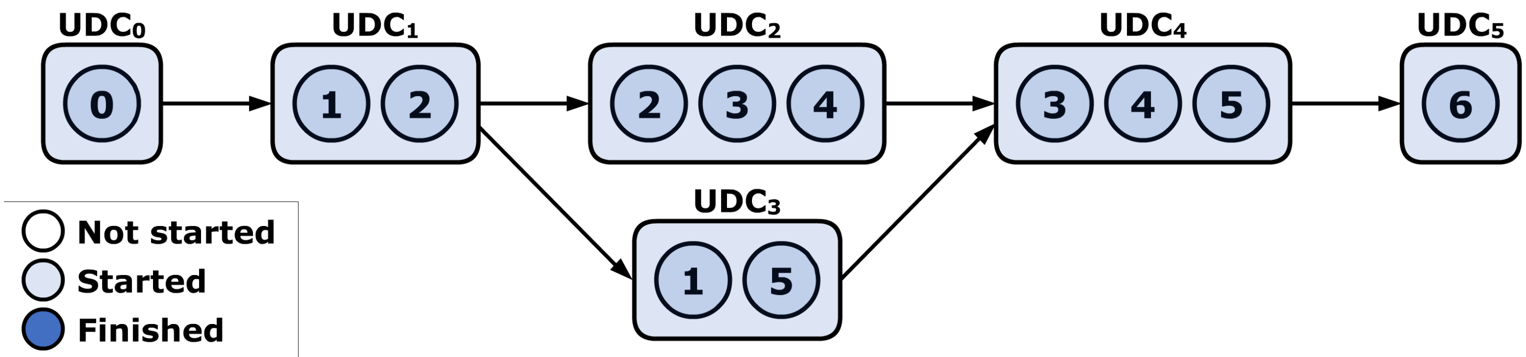

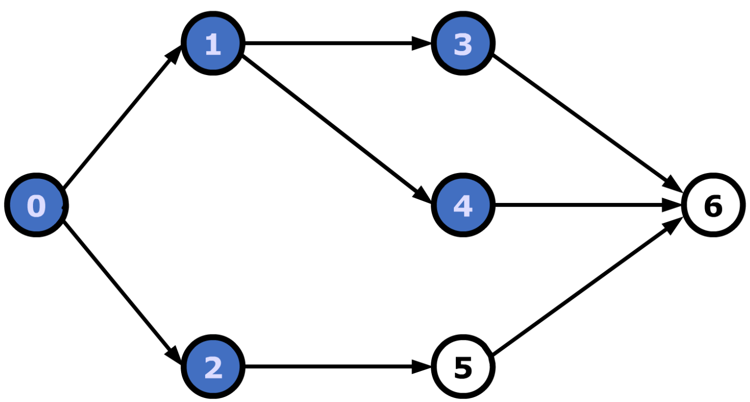

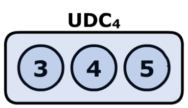

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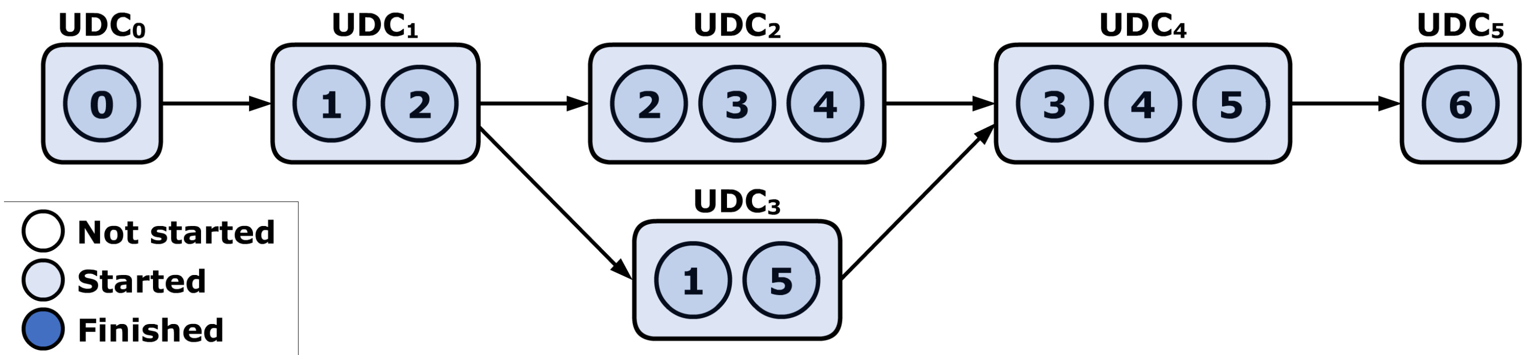

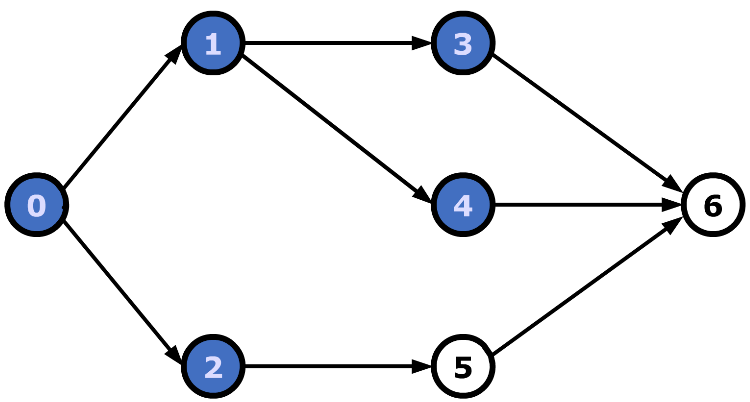

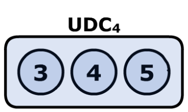

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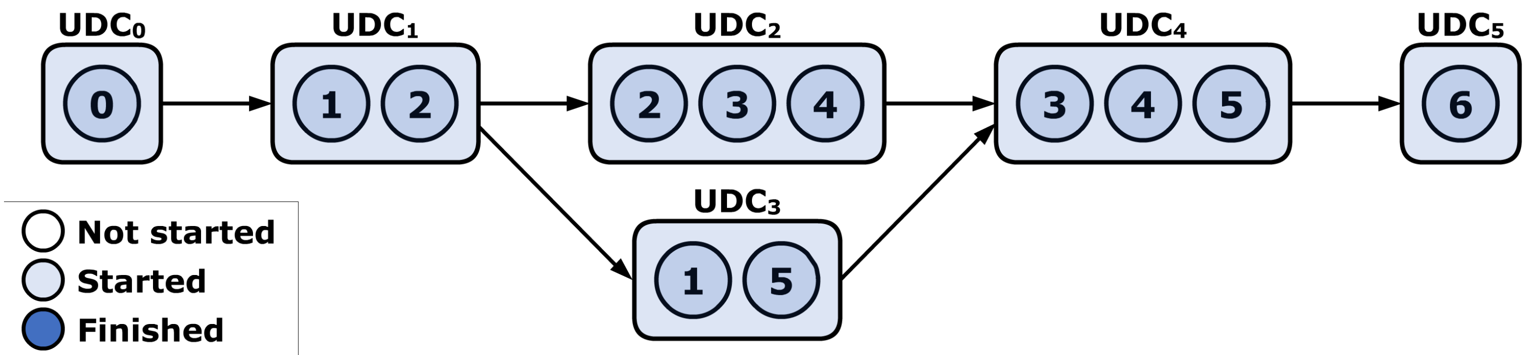

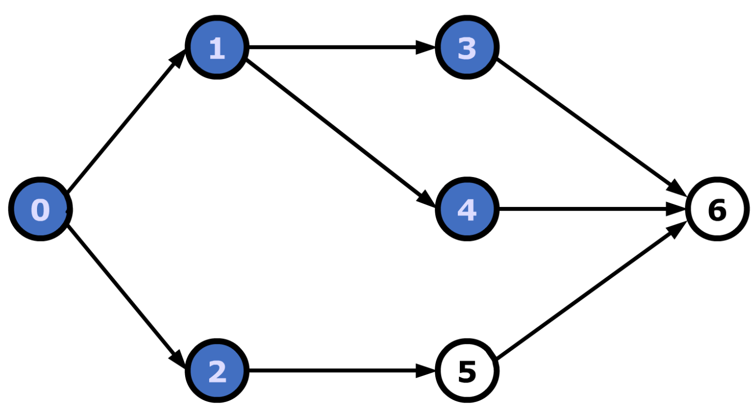

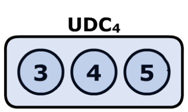

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(2,2,2,2,2,0,0)
↳(2,2,2,2,2,1,0) [58.25M$]

Only decision left is to start activity 5
=> incur cost c_5 = -17M$
=> NPV at state entry = 41.25M$

Model Description:

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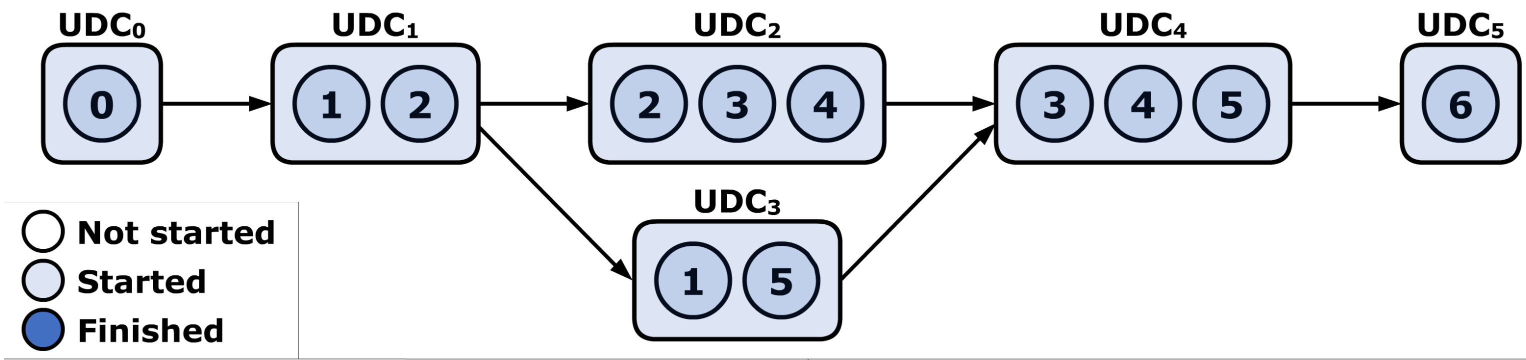

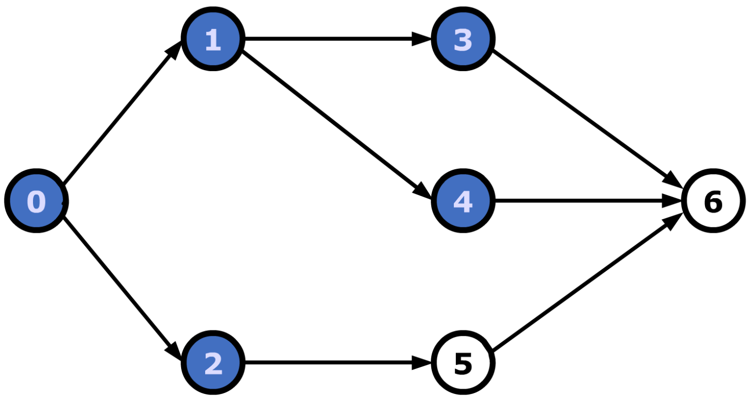

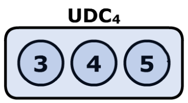

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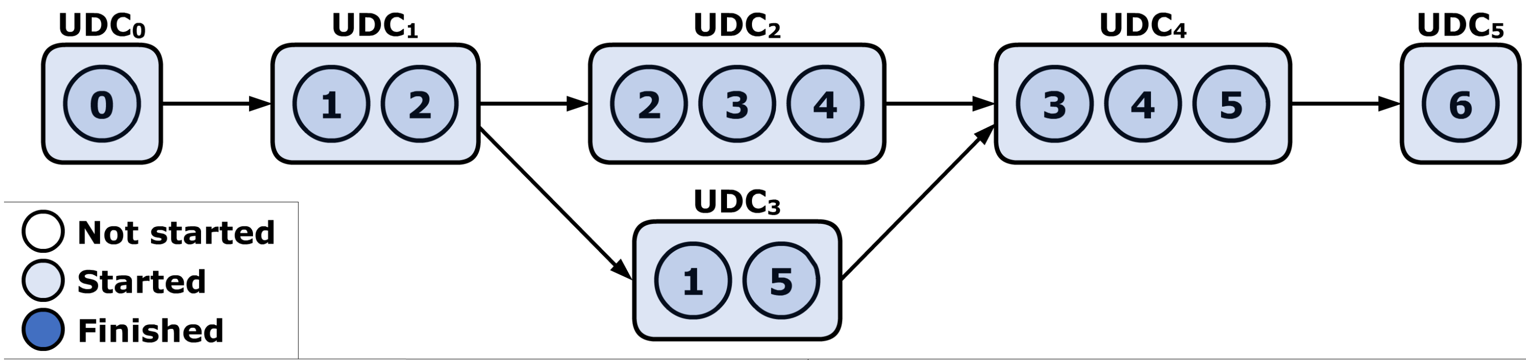

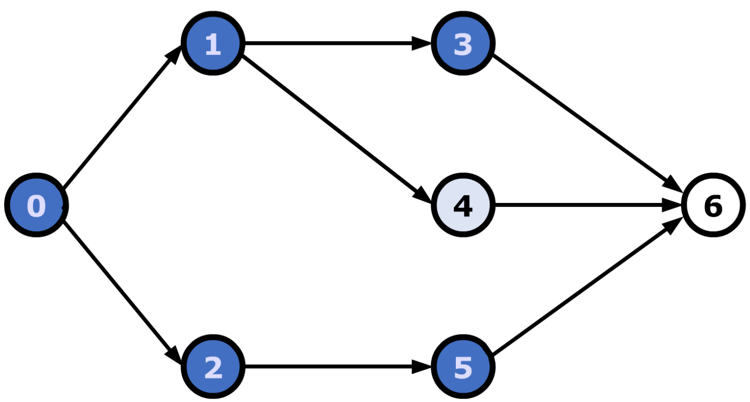

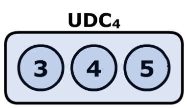

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(2,2,2,2,2,1,0) -> 58.25M$
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Model Description:

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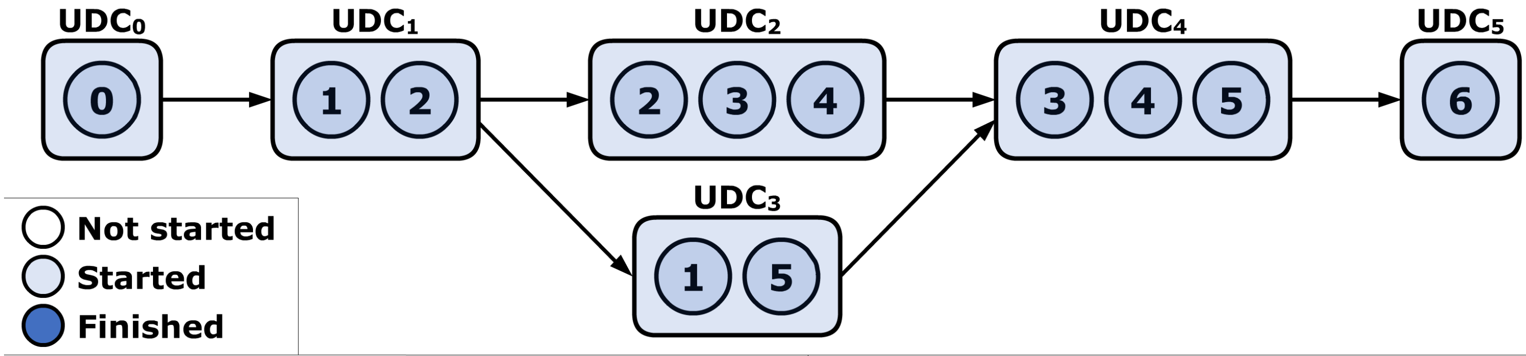

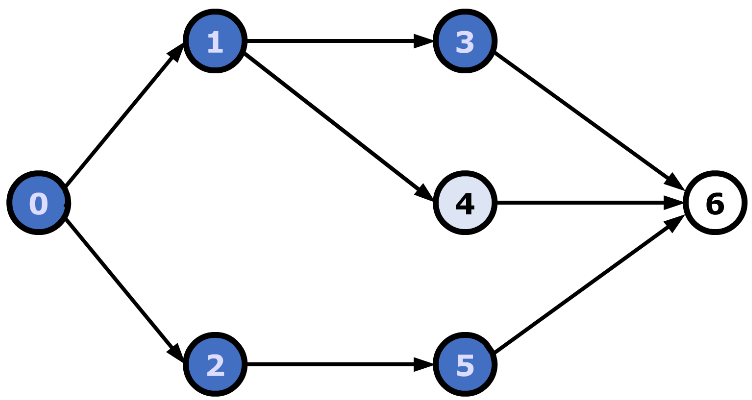

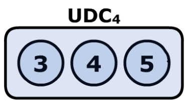

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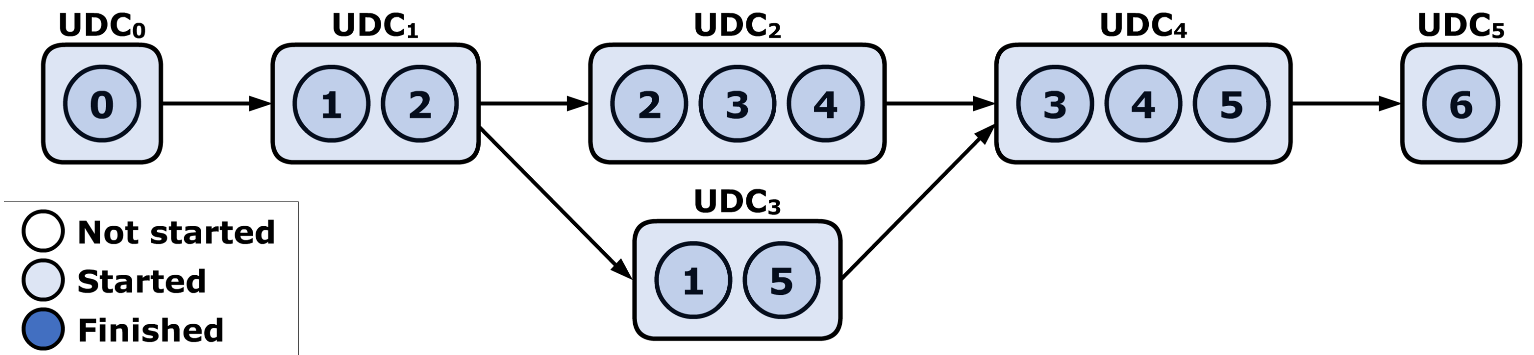
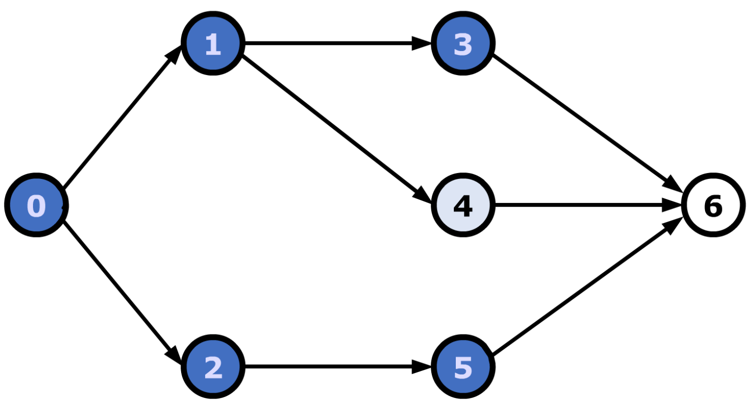
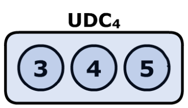
(2,2,2,2,2,1,0) -> 58.25M$
(2,2,2,2,2,0,0) -> 41.25M$
(2,2,2,2,1,2,0)
→ (2,2,2,2,2,2,0) [80.00M$]

Model Description:

Illustration of statespace and backward SDP-recursion







States assigned to UDC:

(2,2,2,2,2,1,0) -> 58.25M$
(2,2,2,2,2,0,0) -> 41.25M$
(2,2,2,2,1,2,0) -> 76.92M$
→ (2,2,2,2,2,2,0) [80.00M$]

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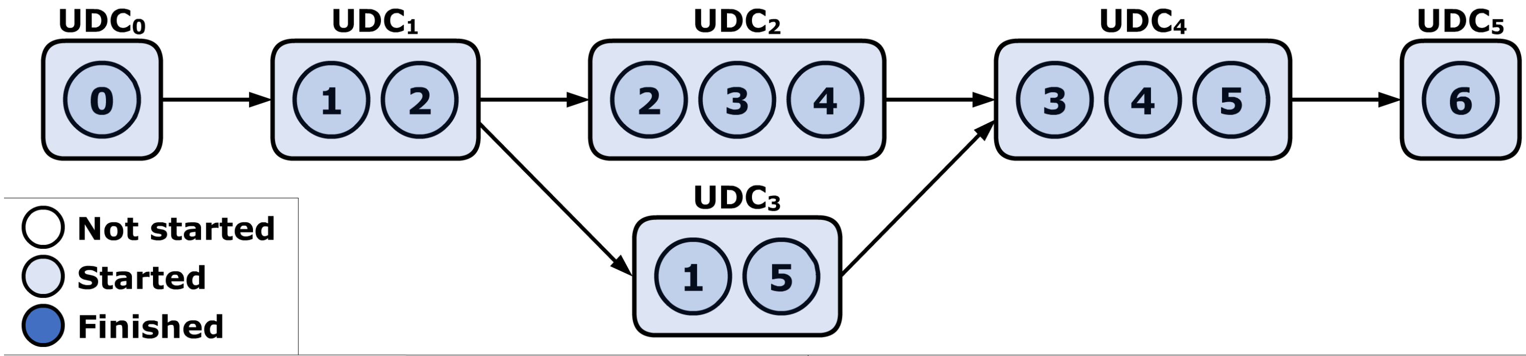

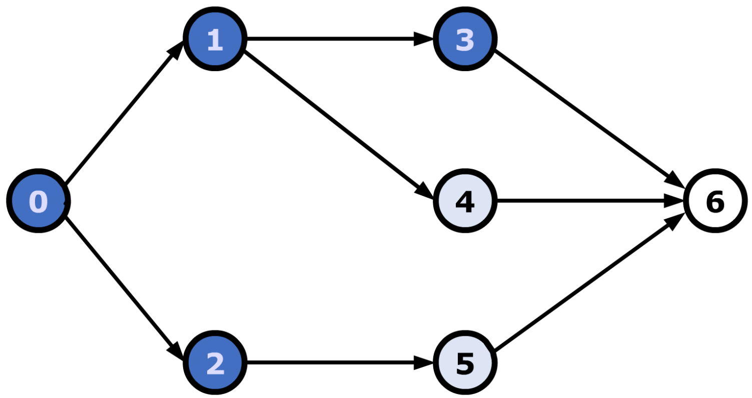

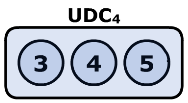

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(2,2,2,2,2,1,0) -> 58.25M$
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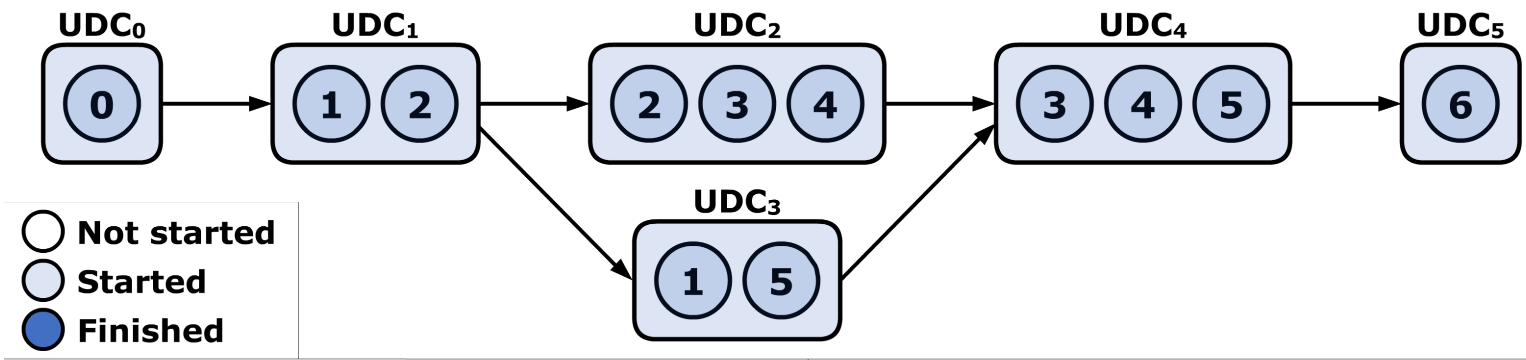
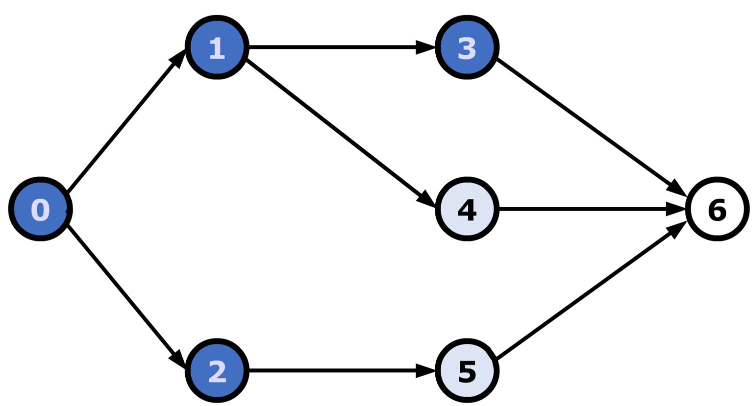
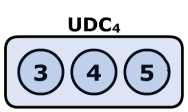
(2,2,2,2,1,2,0) -> 76.92M$
(2,2,2,2,1,1,0)

Model Description:

Illustration of statespace and backward SDP-recursion





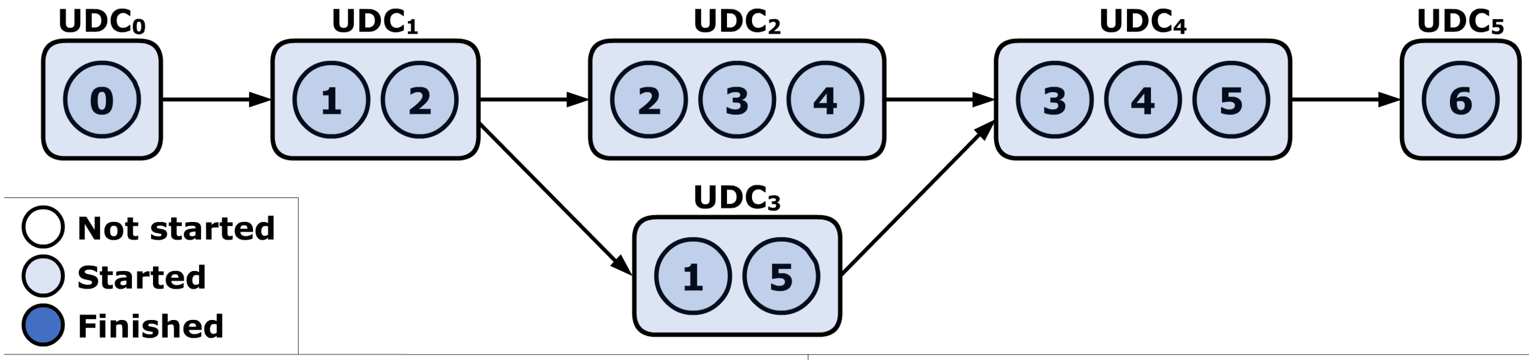


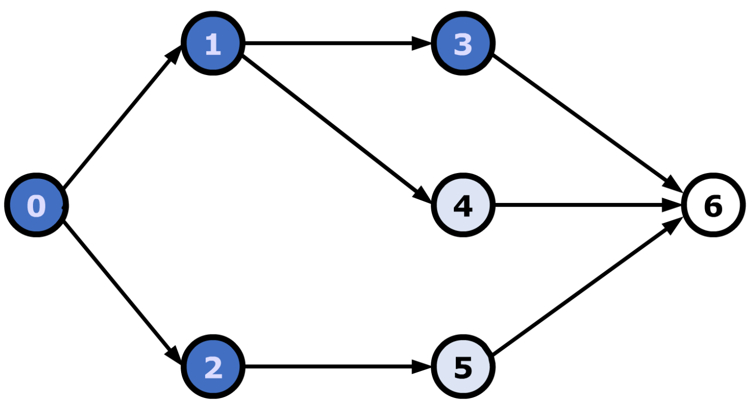
States assigned to UDC:

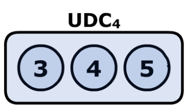
(2,2,2,2,2,1,0) -> 58.25M$
(2,2,2,2,2,0,0) -> 41.25M$
(2,2,2,2,1,2,0) -> 76.92M$
(2,2,2,2,1,1,0)
→(2,2,2,2,2,1,0) [58.25M$]
→(2,2,2,2,1,2,0) [76.92M$]

Model Description:

Illustration of statespace and backward SDP-recursion







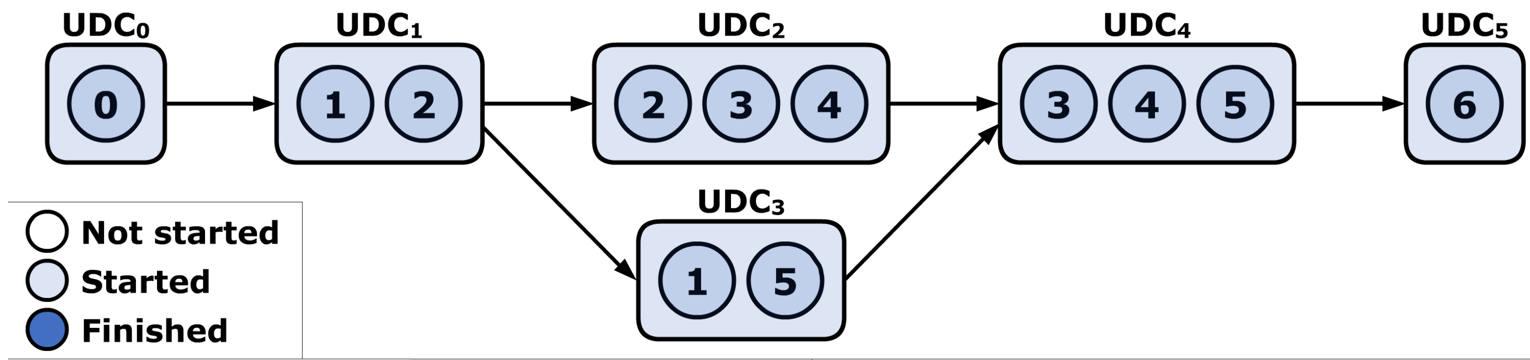
States assigned to UDC:

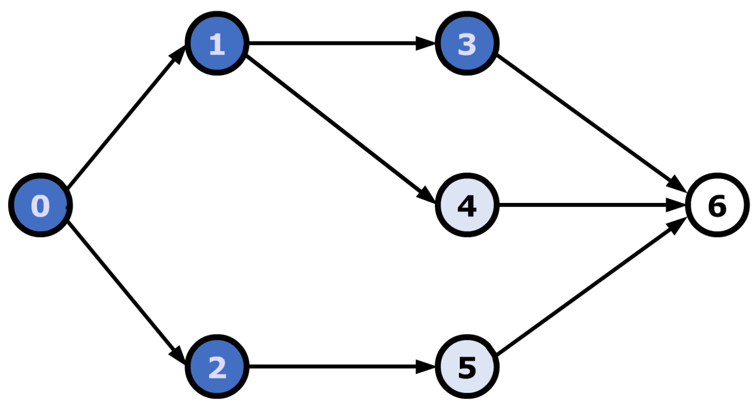
(2,2,2,2,2,1,0) -> 58.25M$
(2,2,2,2,2,0,0) -> 41.25M$
(2,2,2,2,1,2,0) -> 76.92M$
(2,2,2,2,1,1,0)
- **(2,2,2,2,2,1,0) [58.25M$]**
- **(2,2,2,2,1,2,0) [76.92M$]**

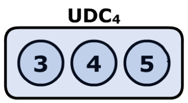
Probability activity finishing first
=> $(1/d_i) \times (\text{SUM}(1/d_i))^{-1}$

Model Description:

Illustration of statespace and backward SDP-recursion







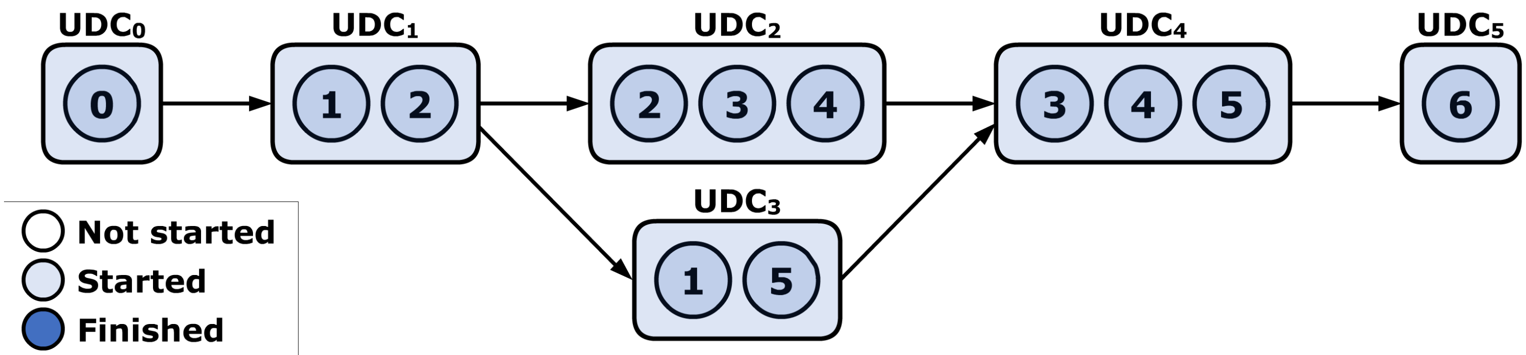
States assigned to UDC:

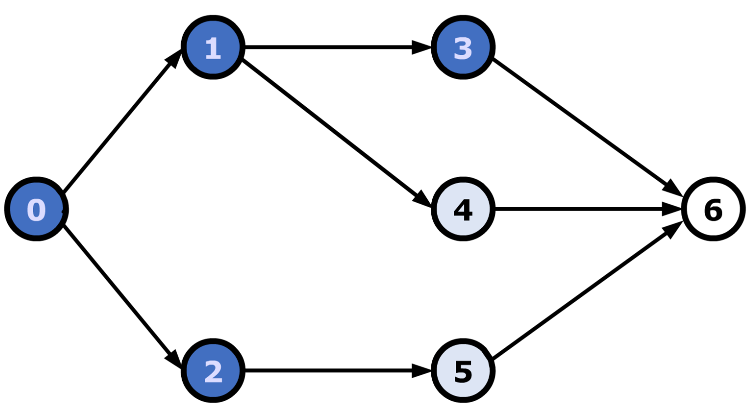
(2,2,2,2,2,1,0) -> 58.25M$
(2,2,2,2,2,0,0) -> 41.25M$
(2,2,2,2,1,2,0) -> 76.92M$
(2,2,2,2,1,1,0)
43% → (2,2,2,2,2,1,0) [58.25M$]
57% → (2,2,2,2,1,2,0) [76.92M$]

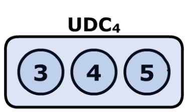
Probability activity finishing first
=> $(1/d_i)\times(\text{SUM}(1/d_i))^{-1}$

Model Description:

Illustration of statespace and backward SDP-recursion







States assigned to UDC:

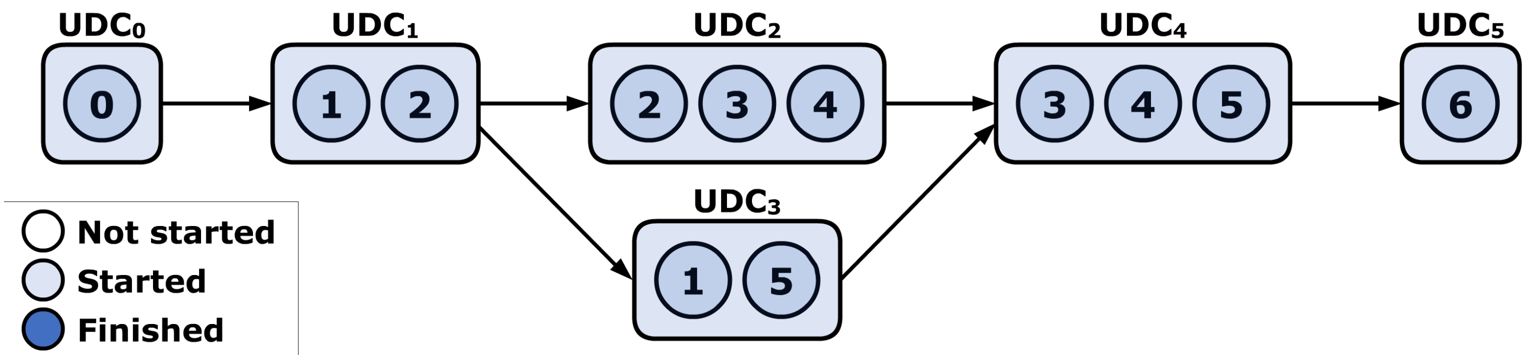
(2,2,2,2,2,1,0) -> 58.25M$
(2,2,2,2,2,0,0) -> 41.25M$
(2,2,2,2,1,2,0) -> 76.92M$
(2,2,2,2,1,1,0) -> 56.96M$

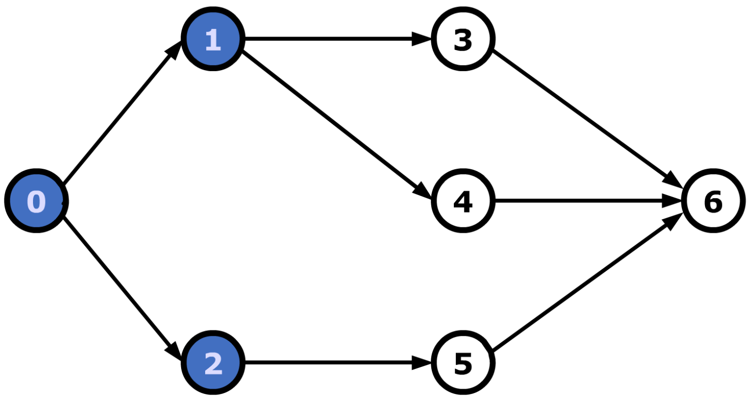
43% → (2,2,2,2,2,1,0) [58.25M$]
57% → (2,2,2,2,1,2,0) [76.92M$]

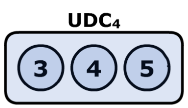
Probability activity finishing first
=> $(1/d_i)\times(\mathrm{SUM}(1/d_i))^{-1}$

Model Description:

Illustration of statespace and backward SDP-recursion





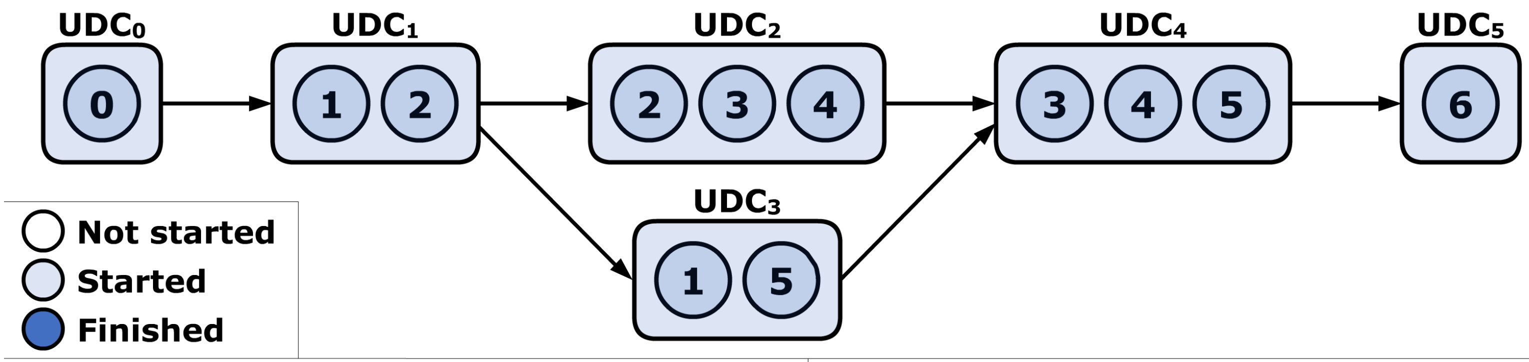


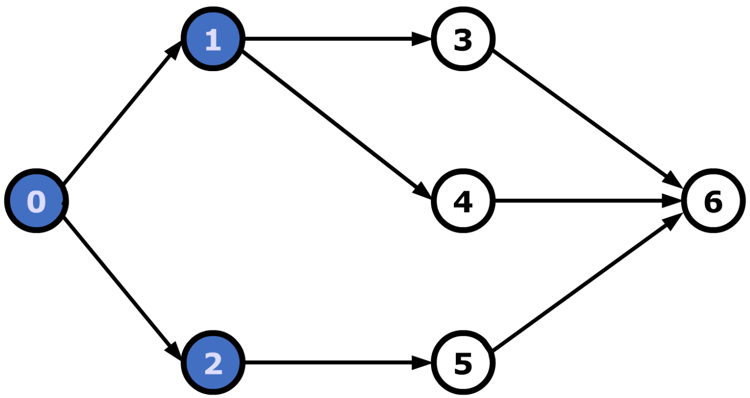
States assigned to UDC:

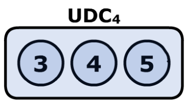
(2,2,2,0,0,0,0)

Model Description:

Illustration of statespace and backward SDP-recursion







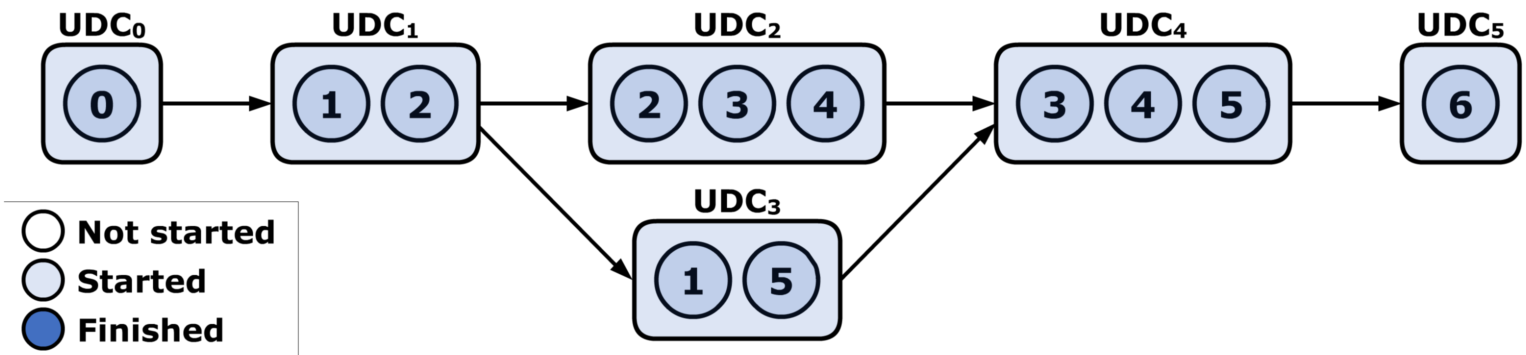
States assigned to UDC:

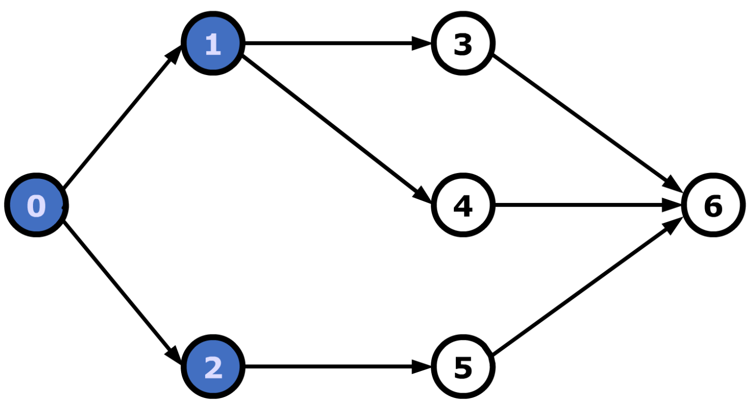
(2,2,2,0,0,0,0)

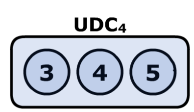
- **(2,2,2,1,0,0,0) [21.51M$]**
- **(2,2,2,0,1,0,0) [17.46M$]**
- **(2,2,2,0,0,1,0) [18.26M$]**
- **(2,2,2,1,1,0,0) [17.46M$]**
- **(2,2,2,1,0,1,0) [18.26M$]**
- **(2,2,2,0,1,1,0) [14.26M$]**
- **(2,2,2,1,1,1,0) [14.17M$]**

Model Description:

Illustration of statespace and backward SDP-recursion





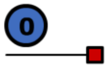


States assigned to UDC:

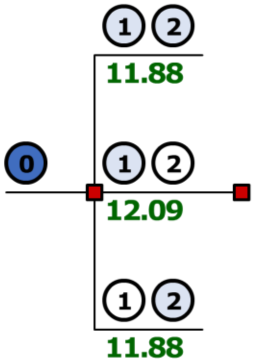
(2,2,2,0,0,0,0) -> 21.51M$

- **(2,2,2,1,0,0,0) [21.51M$]**
- (2,2,2,0,1,0,0) [17.46M$]
- (2,2,2,0,0,1,0) [18.26M$]
- (2,2,2,1,1,0,0) [17.46M$]
- (2,2,2,1,0,1,0) [18.26M$]
- (2,2,2,0,1,1,0) [14.26M$]
- (2,2,2,1,1,1,0) [14.17M$]

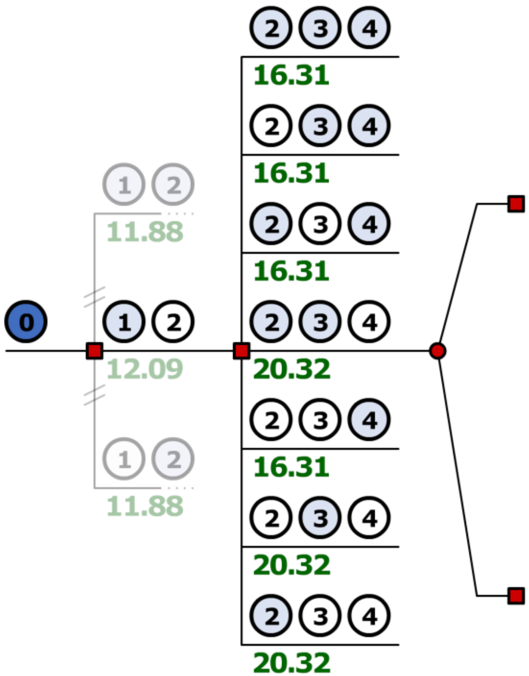
Model Description:
Illustration of the optimal policy



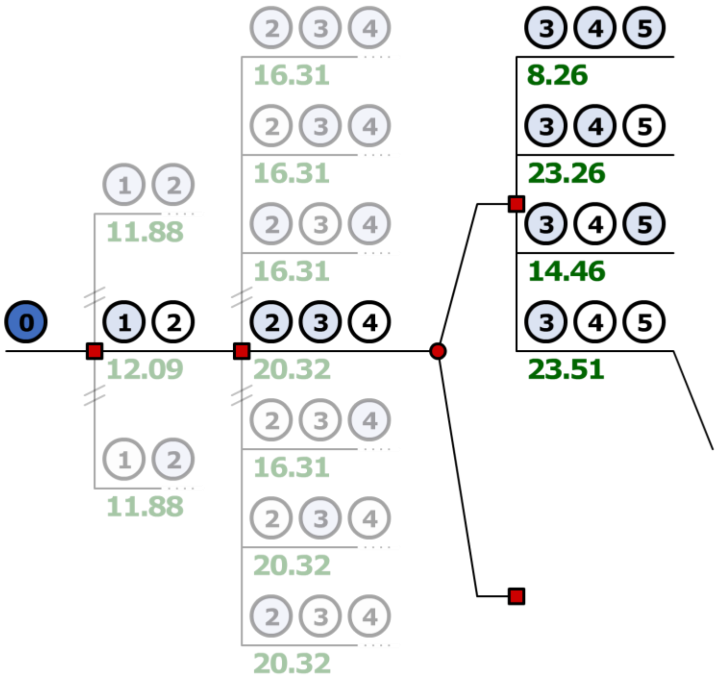
Model Description:
Illustration of the optimal policy



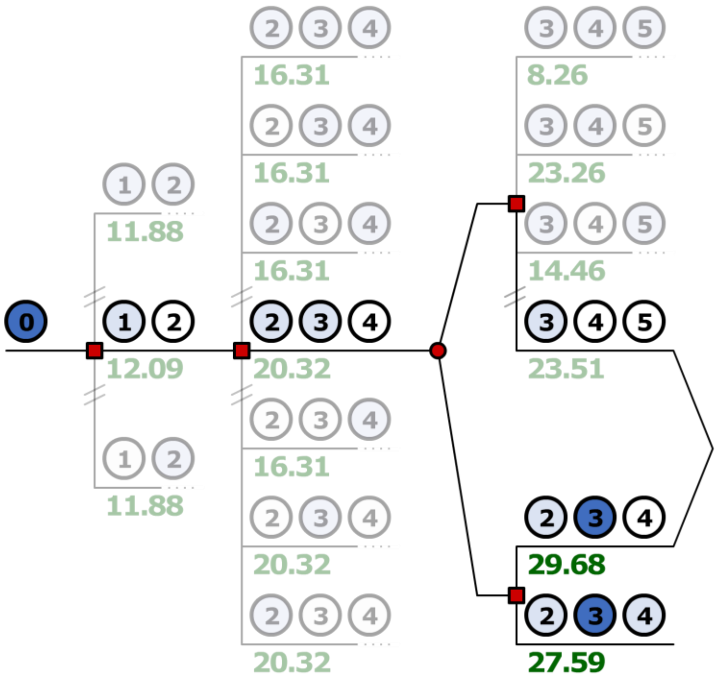
Model Description:
Illustration of the optimal policy



Model Description:
Illustration of the optimal policy

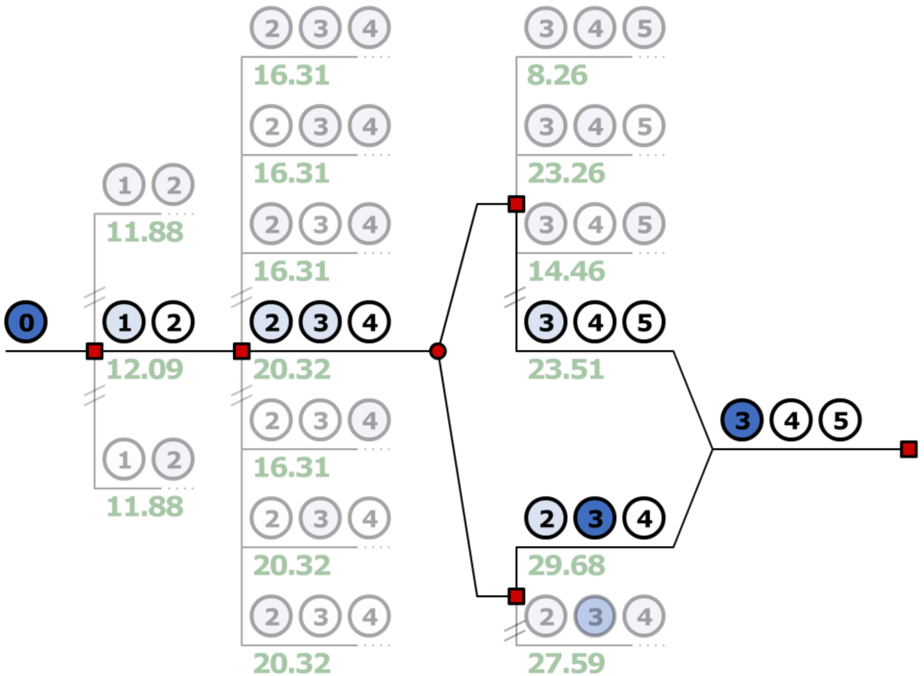


Model Description:
Illustration of the optimal policy

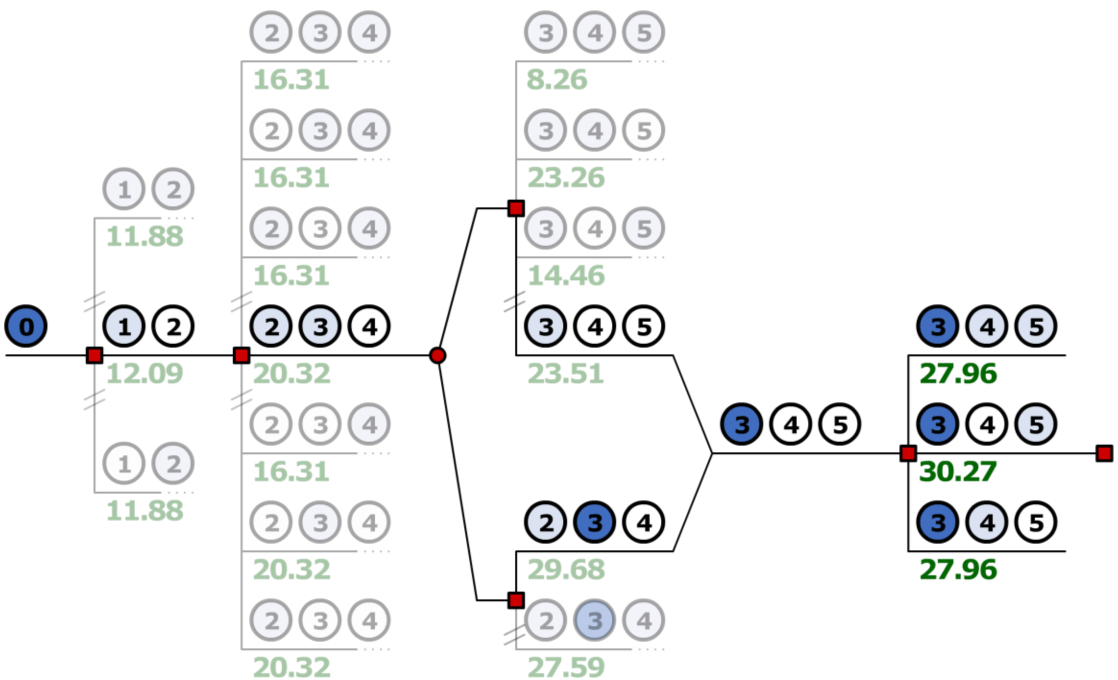


Model Description:

Illustration of the optimal policy

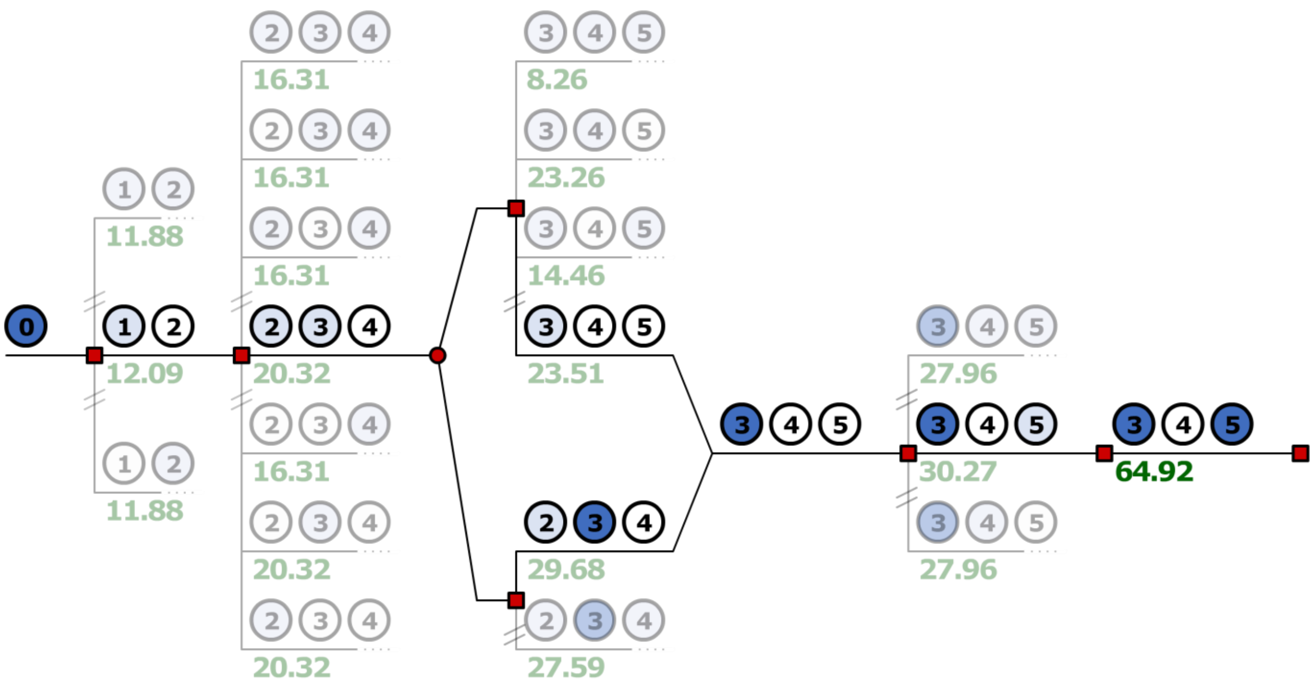


Model Description:
Illustration of the optimal policy

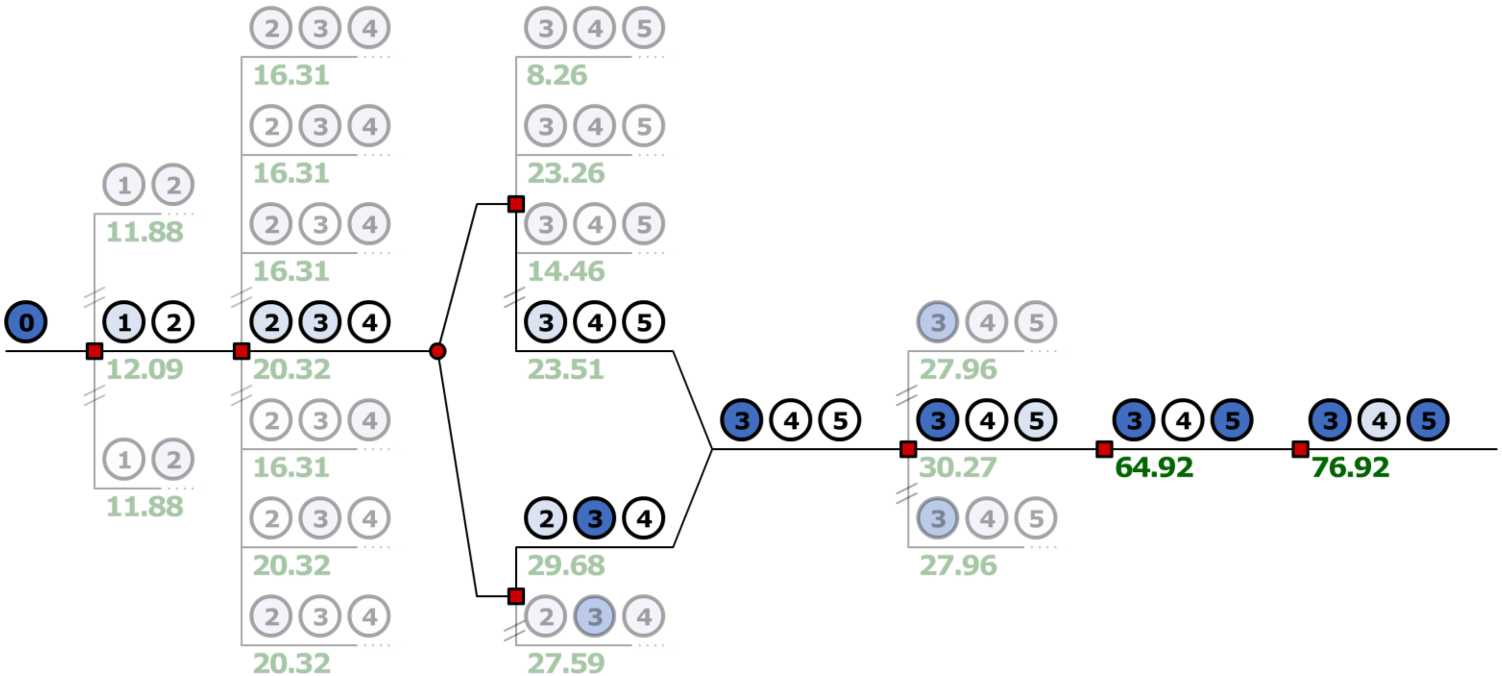


Model Description:

Illustration of the optimal policy

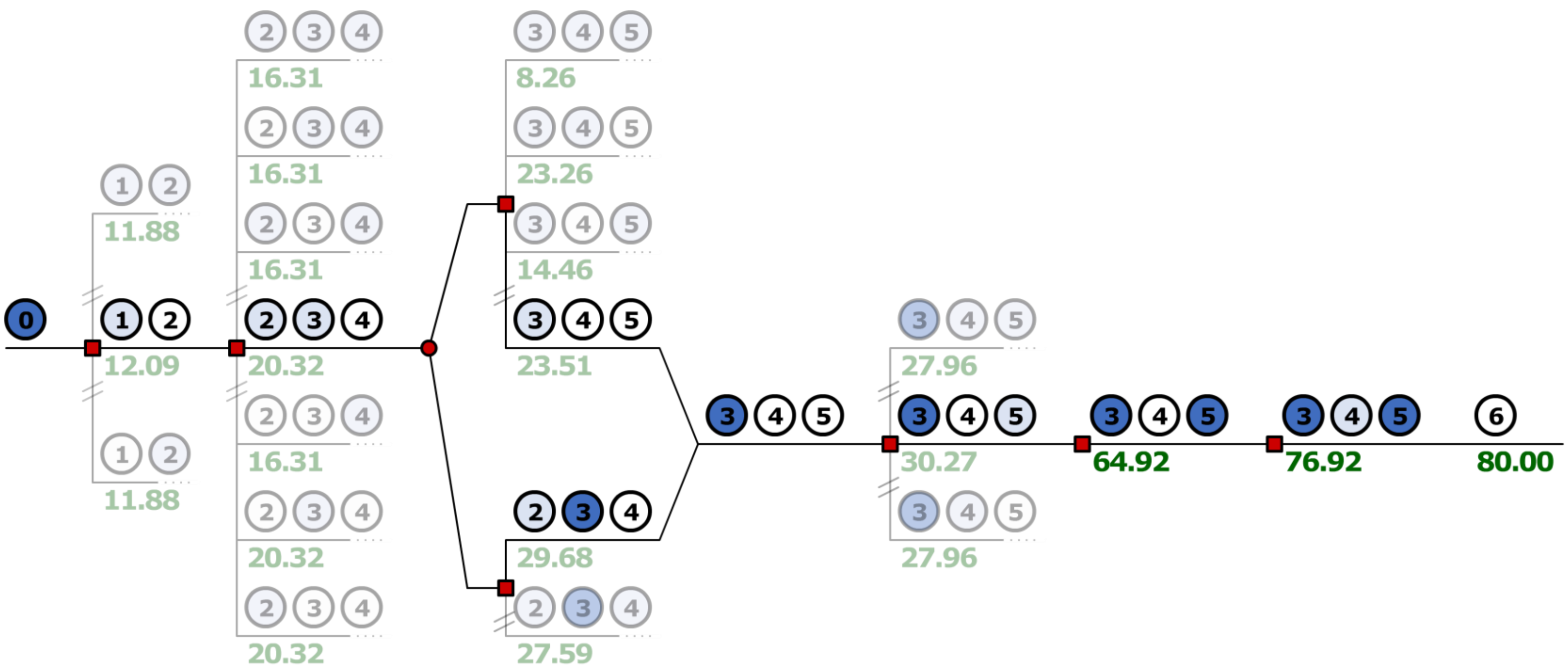


Model Description:
Illustration of the optimal policy



Model Description:

Illustration of the optimal policy



Computational performance (seconds):

AMD Athlon (1.8GHz) – 2048MB RAM

	Number of networks analyzed			Average CPU-time		
n	*OS* = 0.8	*OS* = 0.6	*OS* = 0.4	*OS* = 0.8	*OS* = 0.6	*OS* = 0.4
10	30	30	30	0,00	0,00	0,00
20	30	30	30	0,00	0,03	0,90
30	30	30	30	0,01	0,65	53,22
40	30	30	29	0,06	13,34	4.288,00
50	30	30	4	0,28	173,18	99.381,00
60	30	30		1,29	4.053,00	
70	30	22		5,39	33.247,00	
80	30	9		19,21	115.455,00	
90	30			86,96		
100	30			301,66		
110	30			1.777,00		
120	30			19.245,00		

OS: Order strength; a measure of network density

Conclusions:
Contribution & future research

- Contribution: we develop a model that incorporates:
  - Stochastic activity durations
  - NPV-objective
  - Activity failure
  - Good computational performance (networks of 120 activities are solved to optimality)
- Future research:
  - Modular projects
  - General durations using Phase-Type distributions
  - Resources
  - Activity delay