Project scheduling with alternative technologies: incorporating varying activity duration variability

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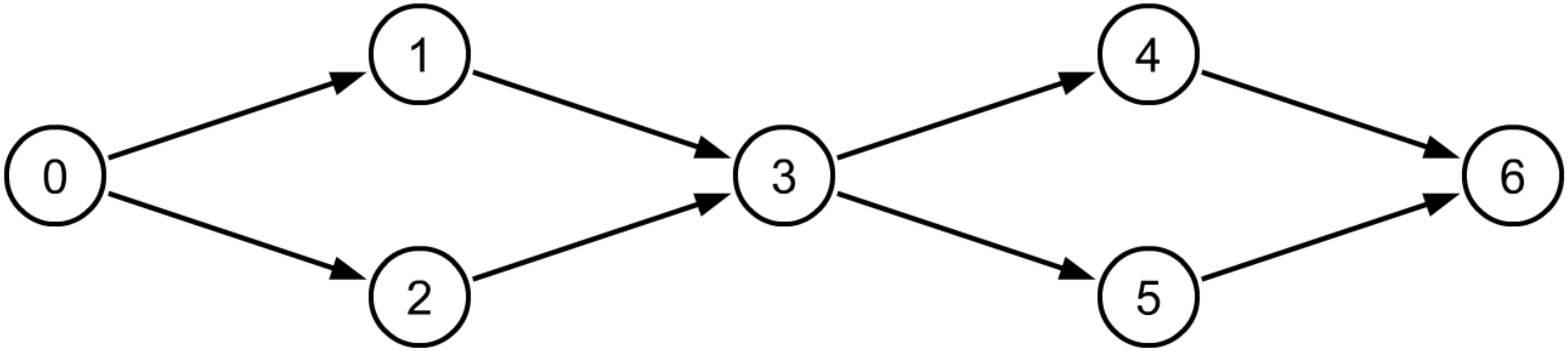
Introduction

- A project is a set of precedence-related activities that need to be completed in order to achieve a specific target
- Our objective is to schedule the activities of a project such that its value is maximized
- We examine how to incorporate the following characteristics:
  - Activity failure
  - Modular completion structure of the project
  - Different levels of variability in the durations of the activities
- Relevant especially for R&D and NPD but also in other sectors: pharmaceuticals, software development, …



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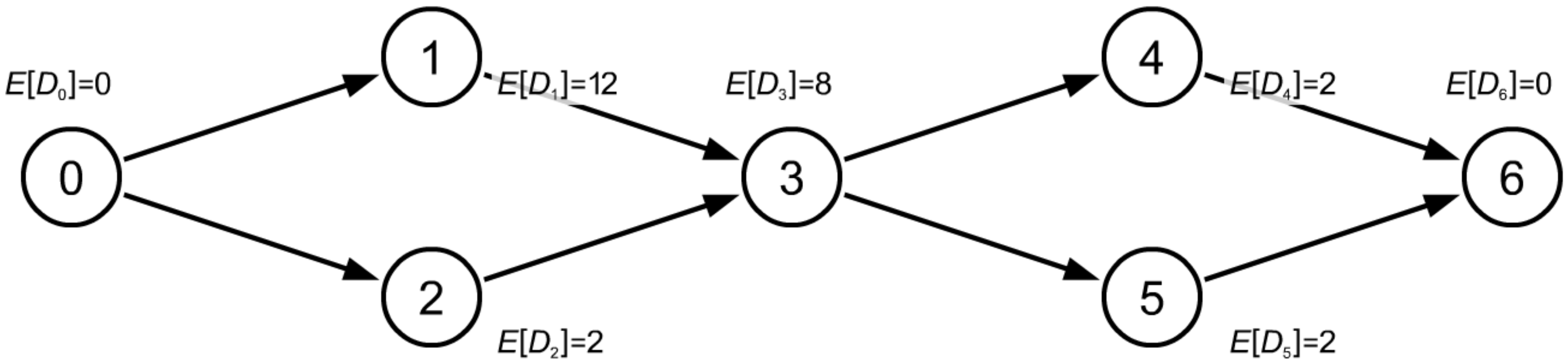

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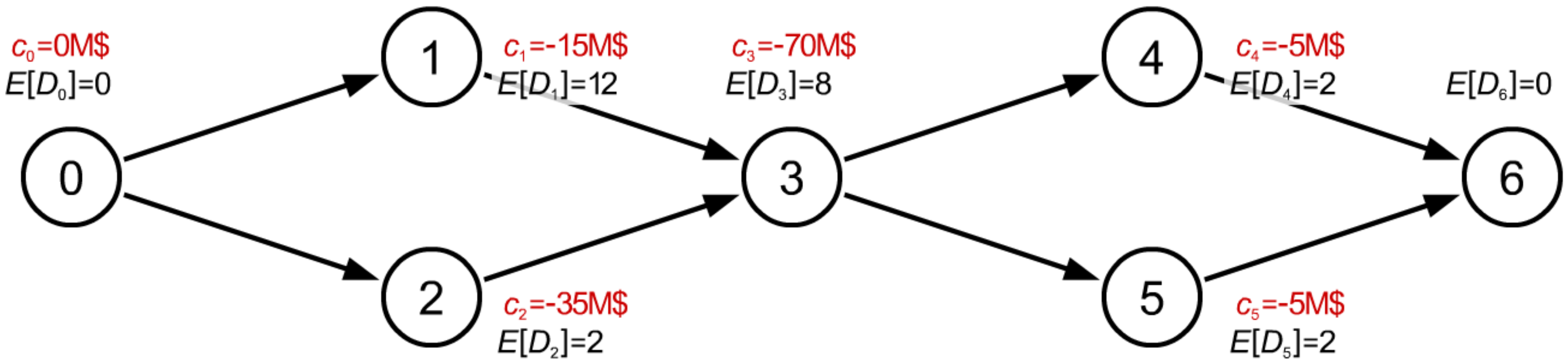

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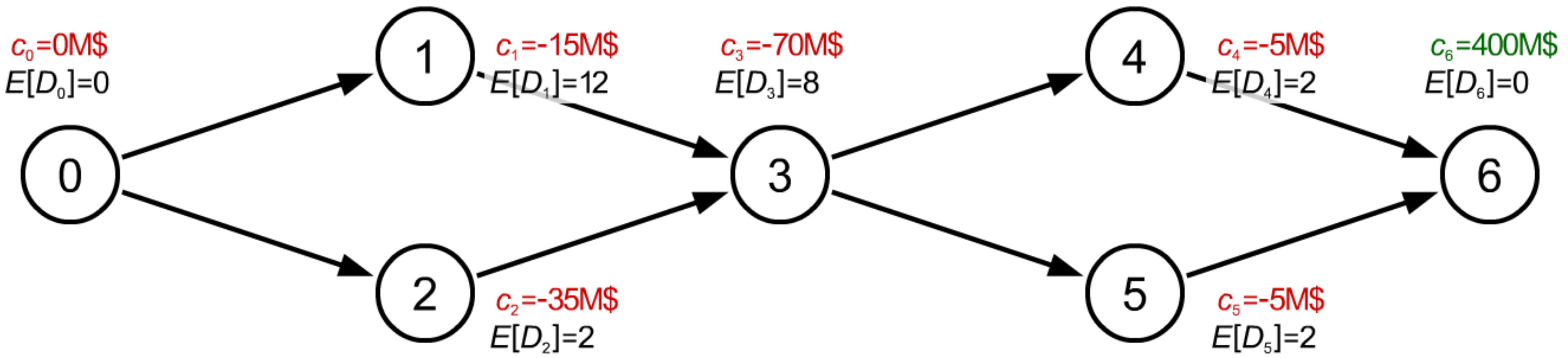

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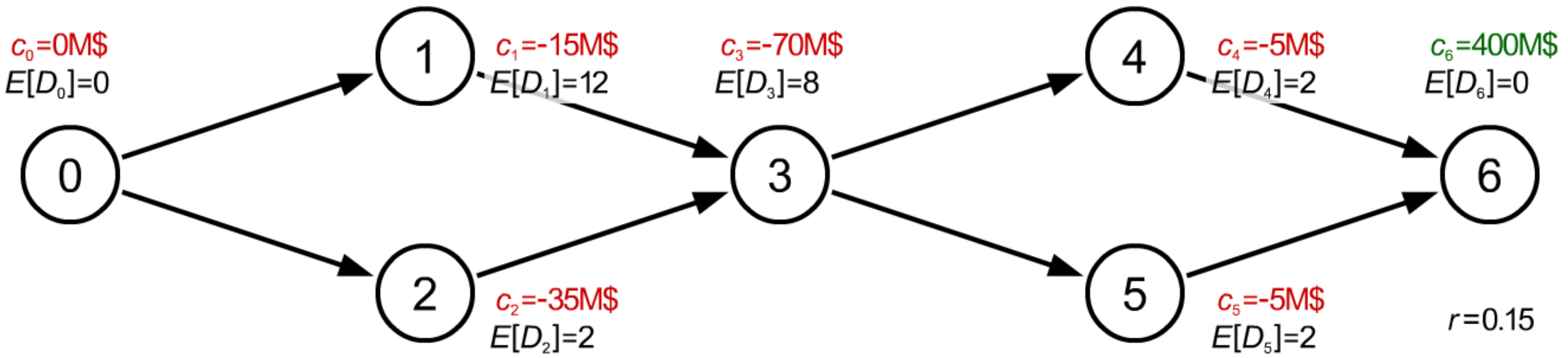

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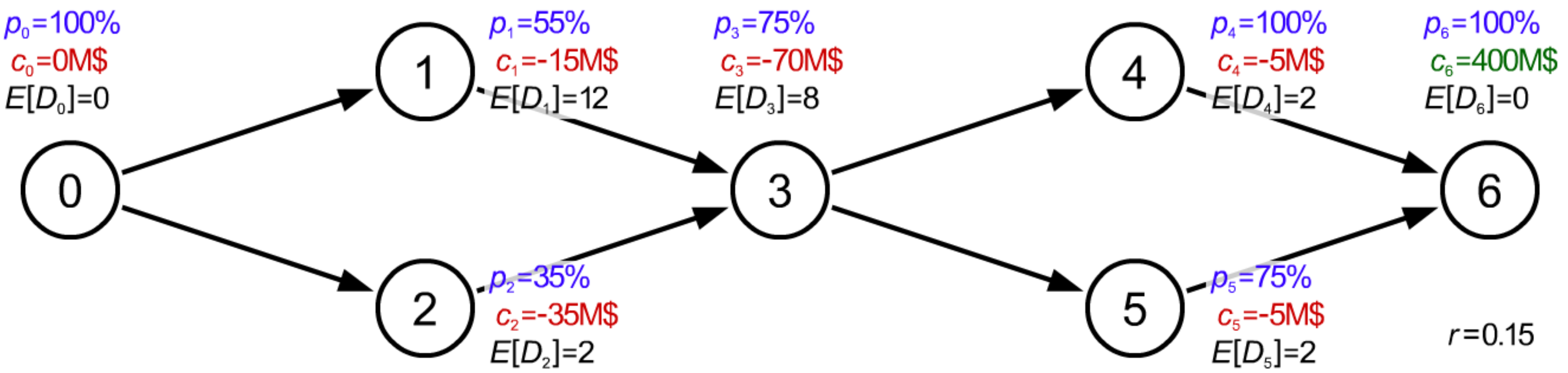

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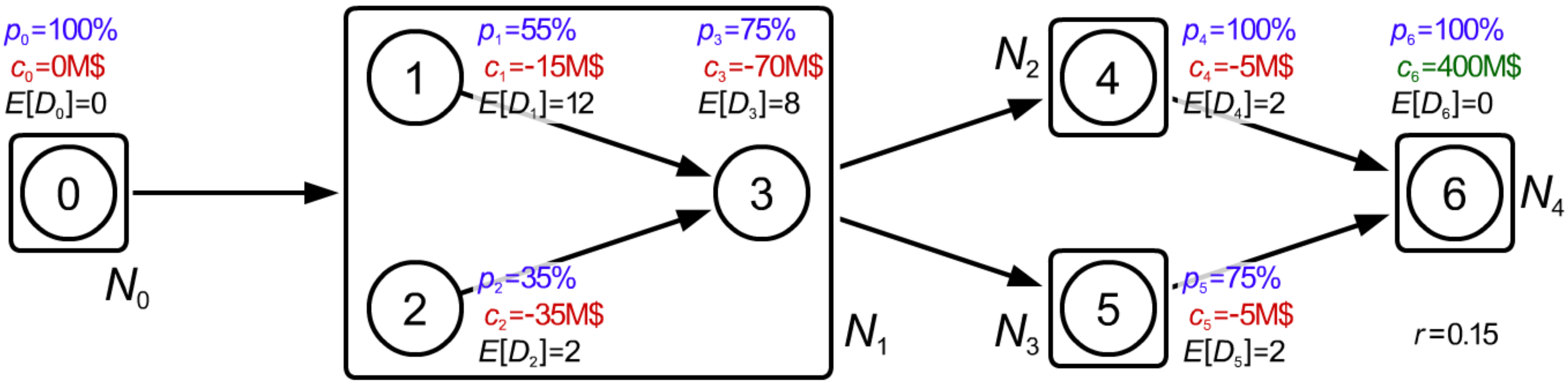

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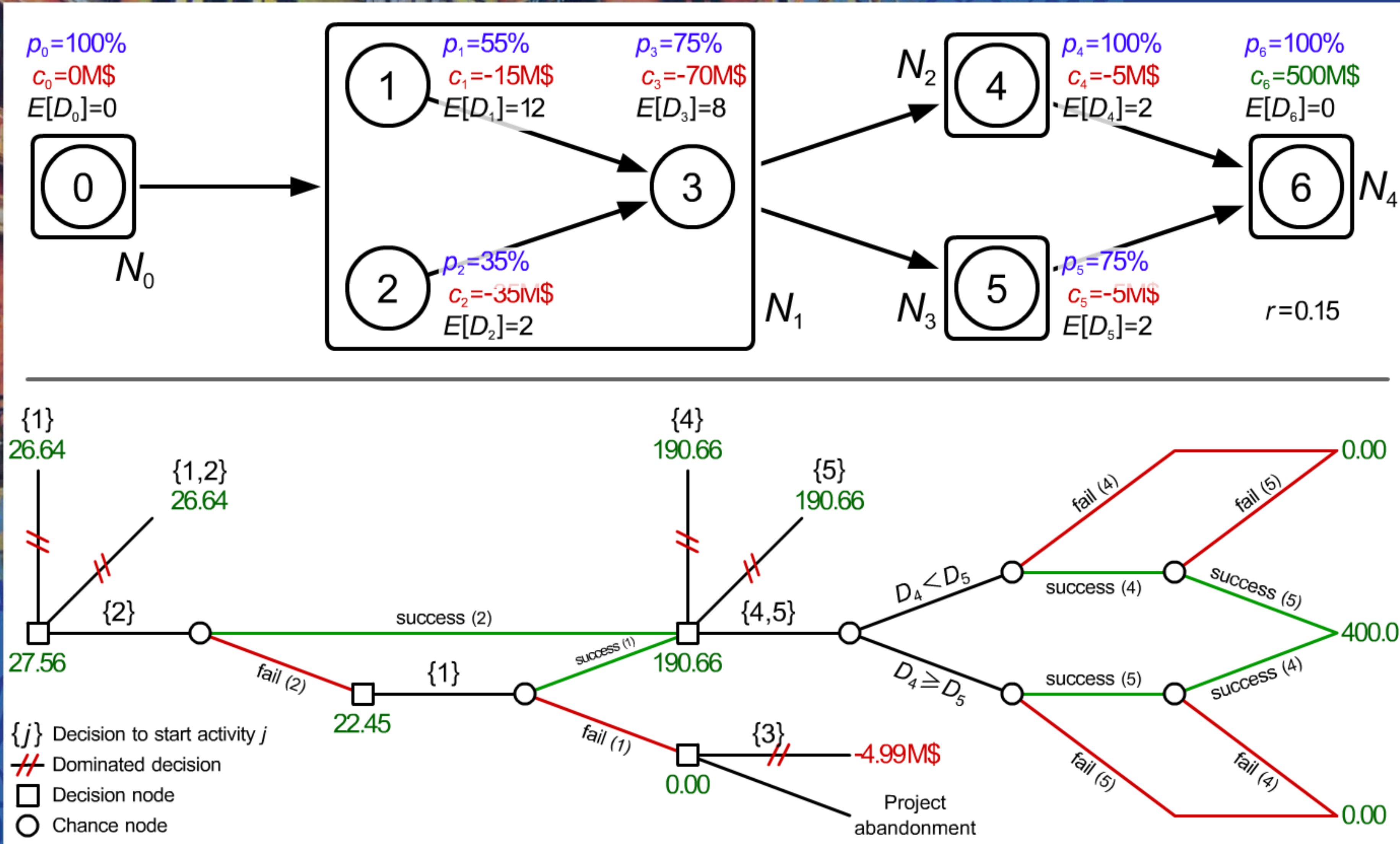
- *m* modules N_i

Solution methodology

We build on earlier work of Creemers et al. (2010)*:

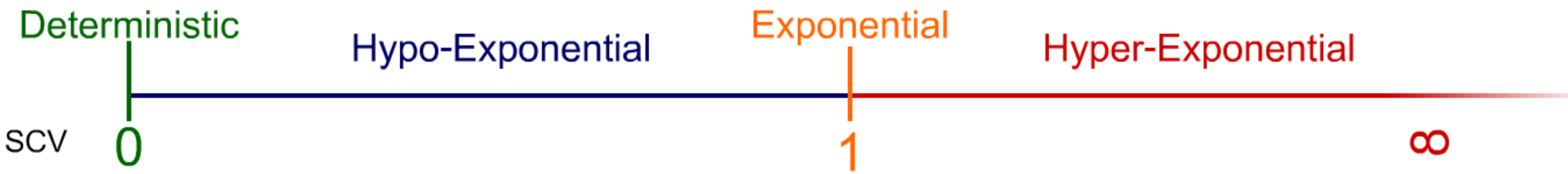
- At any moment in time, the state of each activity *j* can be:
  - Not Started
  - In progress
  - Past (successfully finished, failed or considered redundant because another activity of its module has completed successfully)
- The state of the system is defined by the state of the activities
- Use of Phase-Type distributions to model activity durations
- Use of a Continuous-Time Markov chain to model the statespace
- The optimal eNPV is found using a backward SDP-recursion

*Creemers S, Leus R & Lambrecht M (2010). Scheduling Markovian PERT networks to maximize the net present value. Operations Research Letters, vol. 38, no. 1, pp. 51 - 56.

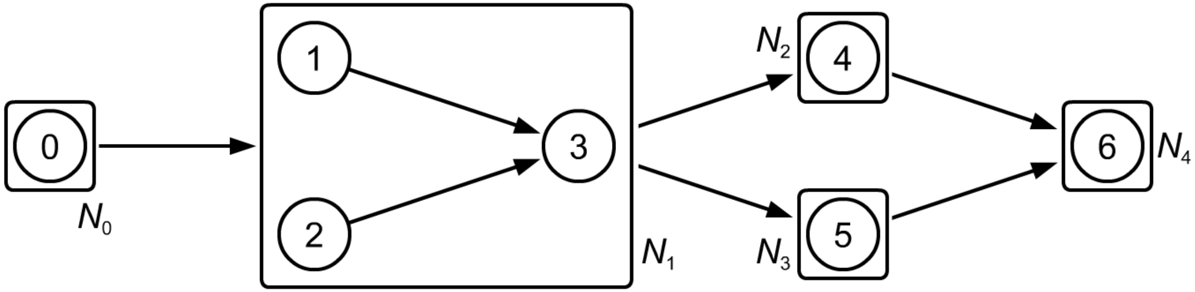

p0=100%
c0=0M$
E[D0]=0
0
N0
1
p1=55%
c1=-15M$
E[D1]=12
2
p2=35%
c2=-35M$
E[D2]=2
3
p3=75%
c3=-70M$
E[D3]=8
N1
N2
4
p4=100%
c4=-5M$
E[D4]=2
N3
5
p5=75%
c5=-5M$
E[D5]=2
6
N4
p6=100%
c6=500M$
E[D6]=0
r=0.15
{1}
26.64
{1,2}
26.64
{2}
27.56
success (2)
fail (2)
22.45
{1}
success (1)
fail (1)
{4}
190.66
{5}
190.66
190.66
{4,5}
{3}
-4.99M$
0.00
Project abandonment
D4<D5
D4≥D5
success (4)
success (5)
fail (4)
fail (5)
success (5)
success (4)
fail (5)
fail (4)
0.00
400.0
0.00
{j} Decision to start activity j
Dominated decision
Decision node
Chance node

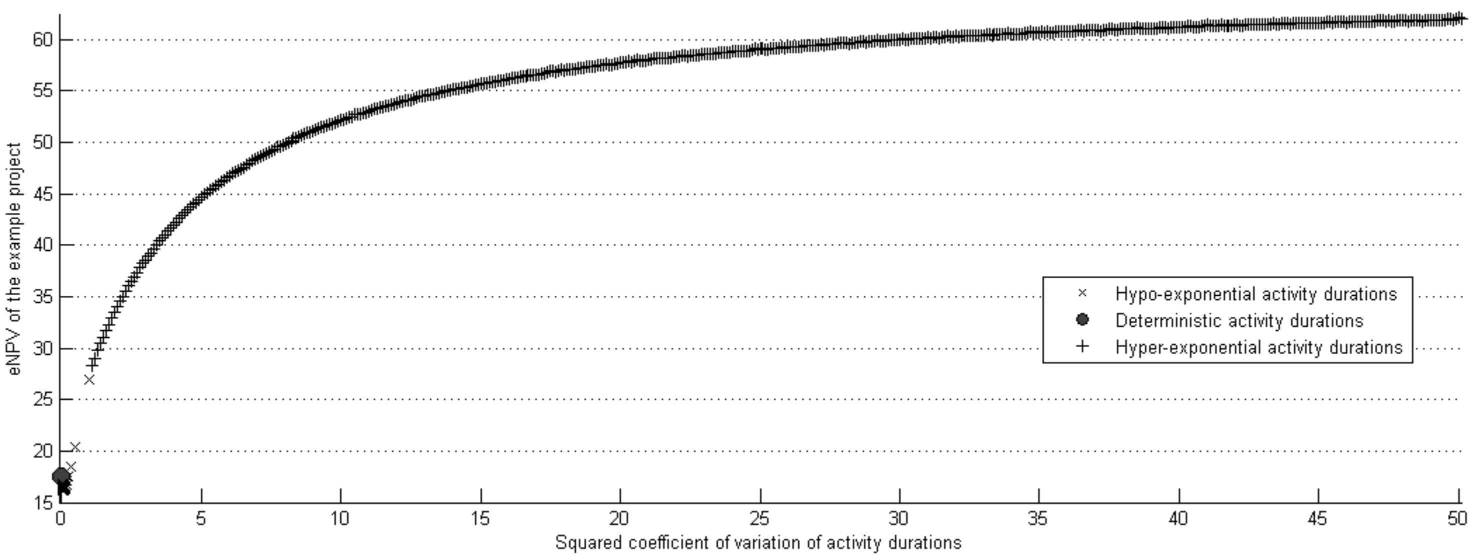
Research Question

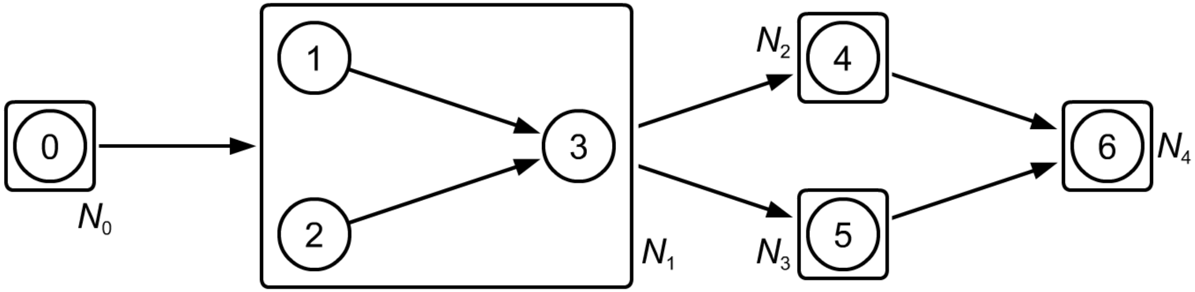
- What is the impact of the variability of activity durations on the eNPV of a project?
- Experimental setup:
  - All activities in the project have the same level of variability
  - Variability is expressed using the Squared Coefficient of Variation (SCV)
  - We observe SCV's ranging from 0 (deterministic) to ∞
  - We use Phase-Type distributions to model the activity durations*

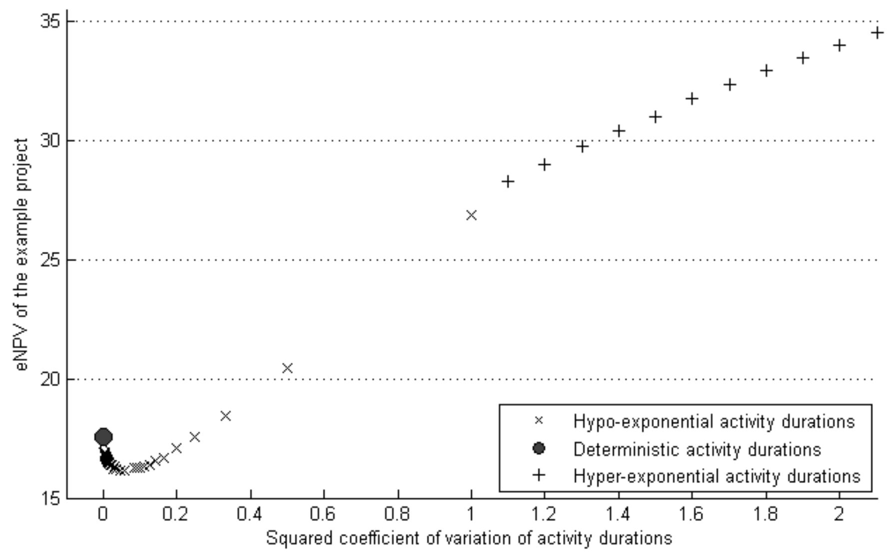


*We adopt the two-moment matching approximation of Creemers S & Lambrecht M (2010). An advanced queueing model to analyze appointment-driven service systems. Computers and Operations Research., vol. 36, no. 10, pp. 2773 - 2785.


0
1
2
3
4
5
6
N0
N1
N2
N3
N4

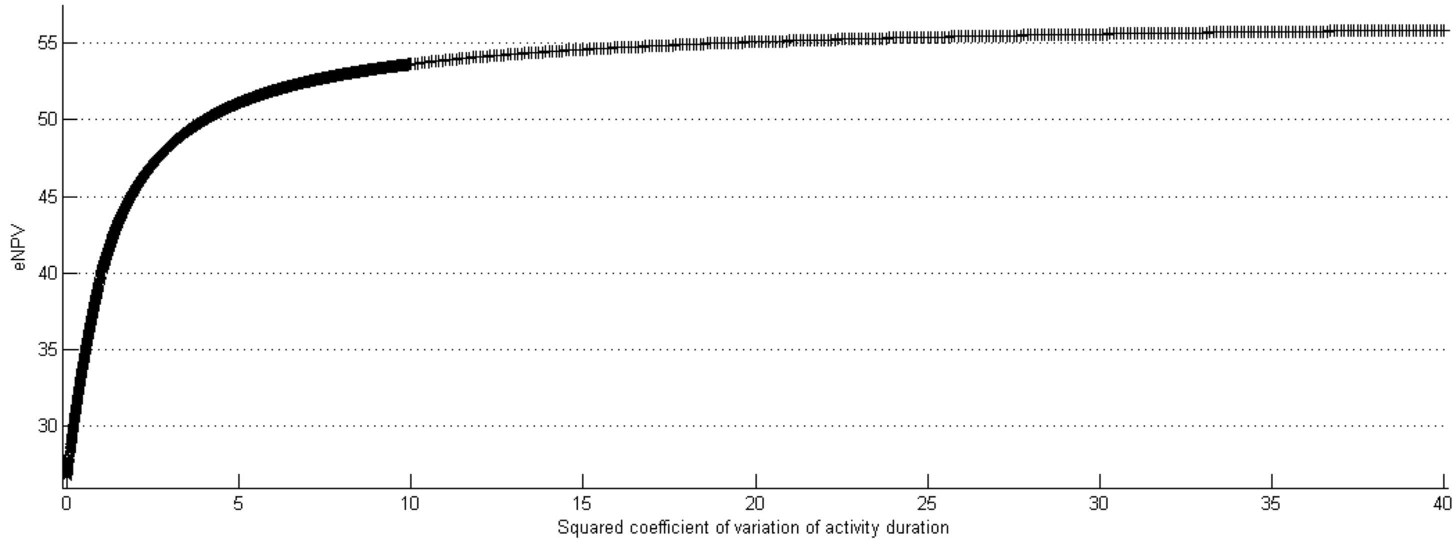

eNPV of the example project
Squared coefficient of variation of activity durations
Hypo-exponential activity durations
Deterministic activity durations
Hyper-exponential activity durations
60
55
50
45
40
35
30
25
20
15
0
5
10
15
20
25
30
35
40
45
50





Preliminary computational results indicate that networks with up to 60 activities can be solved to optimality

Individual activity level



Summary

- We have extended the model of Creemers et al. (2010) to incorporate general activity durations (using Phase-Type distributions) & to allow for modular projects
- We have shown that variability in the duration of activities is not always bad with respect to the NPV of a project
- We have shown that even for a single activity, variability in the duration can be beneficial