Patient Flow Times in the Presence of Outages

A Case Study in a Belgian Hospital

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Problem setting

- Problem setting: healthcare and other services
- Measures of interest:
  - Patient waiting time
- Methodology: queueing theory
  - Focus on manufacturing
  - Healthcare modeling requires distinct approach

A basic queueing system

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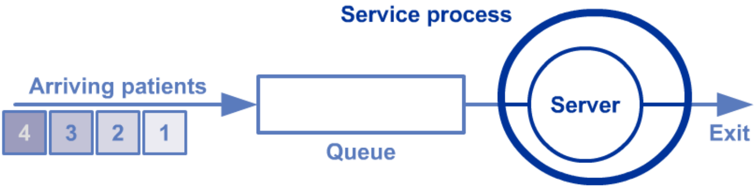

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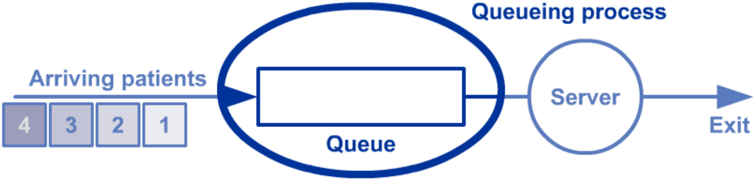

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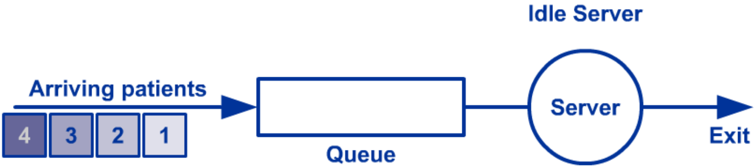

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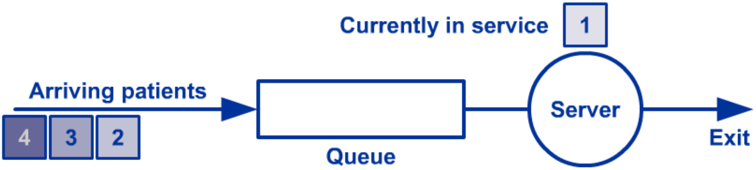

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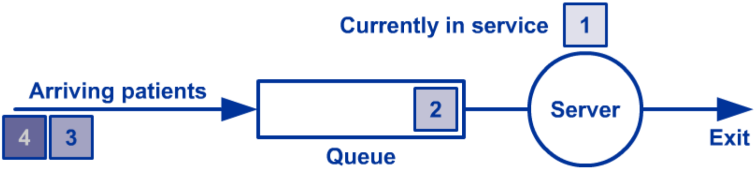

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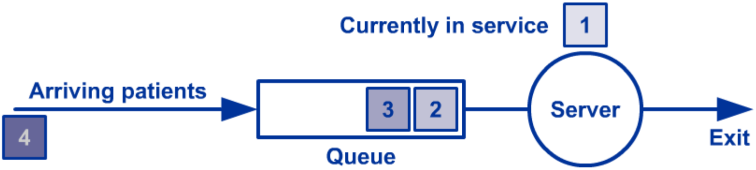

A basic queueing system

Problems in healthcare modeling

- Queue discipline
- Time varying demand
- Waiting creates additional work
- Service outages (absences and interrupts)
- Service epochs
- Reentry at previous workstations
- Probabilistic routing of patients

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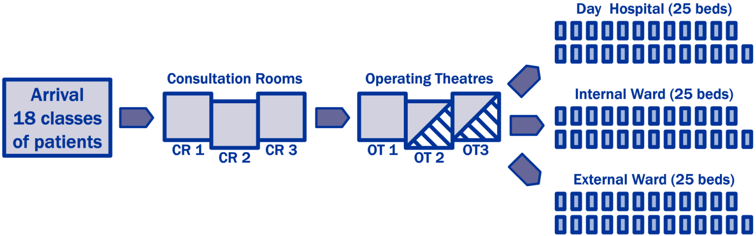
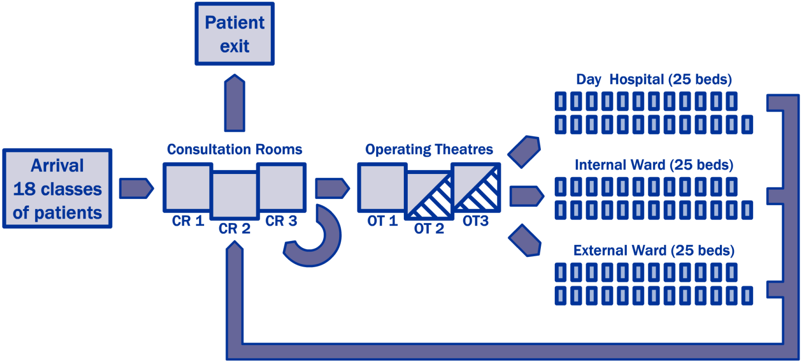
Outline

- Problem setting: orthopaedic department of the Middelheim hospital in Antwerp
- Problem: excessive waiting times and congested waiting list
- Objective: analysis of impact of service outages through capacity and variability analysis
- Methodology: queueing models of the orthopaedic department
- Contribution: development of new expressions to assess the impact of service outages

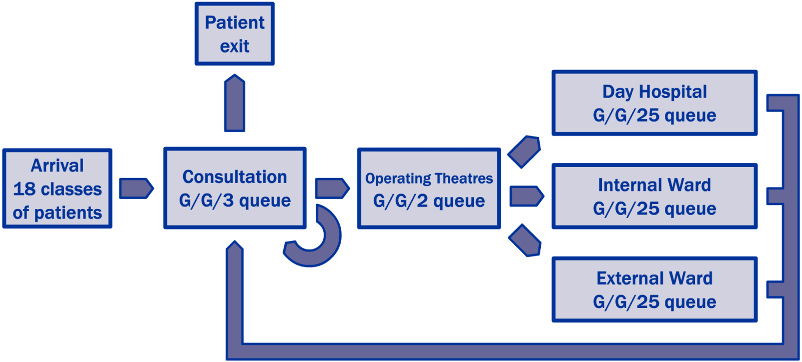
Capacity structure



Patient flow



Queueing network



Service outages

Different types of outages of the service process:

- Nonpreemptive outages (absences)
- Preemptive outages (interrupts)
- Service epochs (server unavailability)

We formulate queueing models taking these outages into account

Nonpreemptive outages

- Interruption of the service process prior to treatment of a patient
- Examples:
  - Absence of medical staff at the beginning of a working shift
  - Setup time of medical facilities (e.g. cleaning, preparation)
- Exact results have been obtained in Hopp and Spearman (2000) under the assumption of a fixed number of patients in between two subsequent outages

Nonpreemptive outages: example

Nonpreemptive outages: example

Preemptive outages

- Interruption of the service process during service itself
- Examples:
  - Emergencies
  - Phone calls, administration, ...
- Exact results have been obtained in Hopp and Spearman (2000) under the assumption of:
  - Exponential time between interrupts
  - Interrupts only occur during the service process itself

Preemptive outages

- Example:

Preemptive outages

- Example:

Preemptive outages

- Example:

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- Example:

- Exact formulation of mean and variance of service times including preemptive outages:

$$\frac{1}{\mu} = \frac{1}{\mu_0}\left(\frac{MTTI}{MTTI + MTTR}\right)$$

$$\sigma^2 = \sigma_0^2\left(\frac{MTTI + MTTR}{MTTI}\right)^2 + \frac{1}{\mu_0}\left(\frac{\sigma_r^2 + MTTR^2}{MTTI}\right)$$

Preemptive outages: generalization

- In healthcare, services may be interrupted during the resolving of a previous interrupt
- Examples:
  - A doctor receiving a phone call during an emergency
  - A doctor who is interrupted by a nurse during a phone call
- We generalize the result of Hopp and Spearman (2000) to include multiple order interrupts

Preemptive outages: generalization

- Example:

Preemptive outages: generalization

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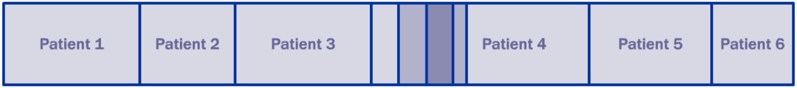
Preemptive outages: generalization

- Example:



Preemptive outages: generalization

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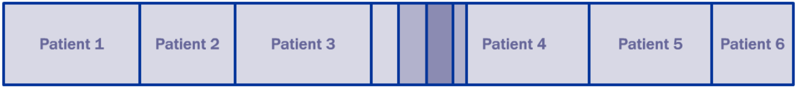

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$$\sigma^2 = \frac{MTTI^2\sigma_0^2 + \frac{1}{\mu_0}\left(MTTI - MTTR\right)\left(\sigma_r^2 + MTTR^2\right)}{\left(MTTI - MTTR\right)^2}$$

Service epochs

- (Healthcare) services take place during predefined time intervals
- Problem: how to combine surgery, consultation and recovery into one model, while all operate on different time scales:
  - Consultation and surgery take place at weekdays during specific hours
  - Recovery is a continuous process
- Solution: rescaling of the service process using an availability concept

Availability

- Rescales the service process in order to fit a predefined uniform time scale (e.g. 24 hours per day, 7 days per week)
- Example: doctor's office with opening hours on Thursday from 6 PM until 8 PM and on Friday from 2 PM until 6 PM
- Availability:

$$A = \frac{6}{168} = \frac{1}{28}$$

Availability

- Rescales the service process in order to fit a predefined uniform time scale (e.g. 24 hours per day, 7 days per week)
- Example: doctor's office with opening hours on Thursday from 6 PM until 8 PM and on Fridays from 2 PM until 6 PM
- Mean and variance of the rescaled service times:

$$\frac{1}{\mu} = \frac{1}{A\mu_0}$$

$$\sigma^2 = \frac{\sigma_0^2}{A^2}$$

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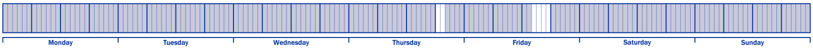

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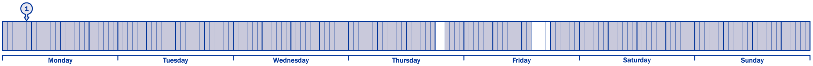

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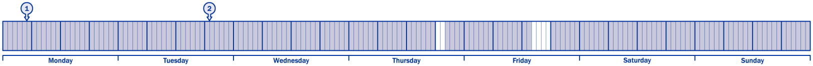

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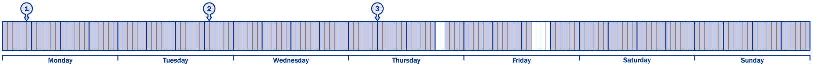

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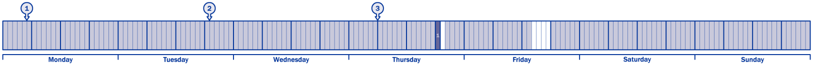

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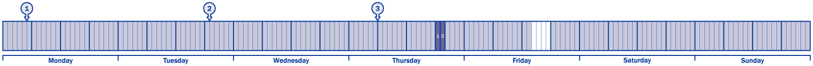

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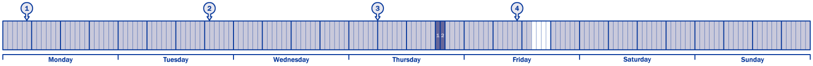

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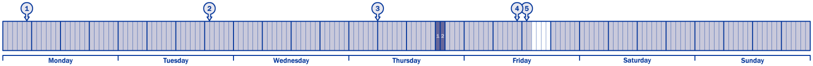

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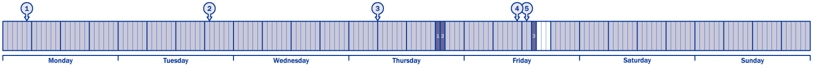

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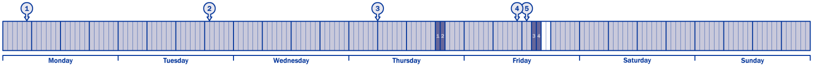

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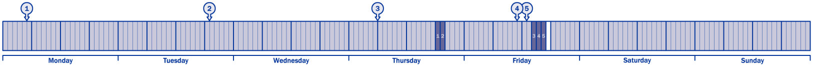

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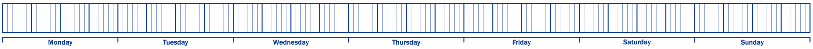

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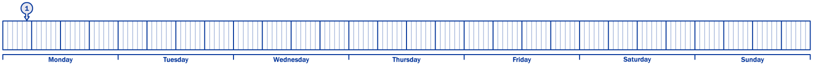

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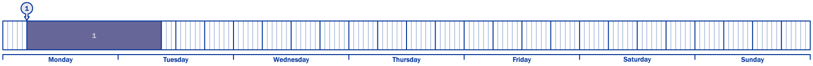

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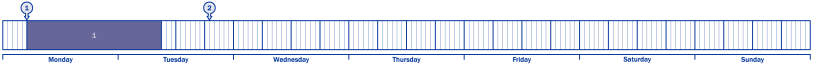

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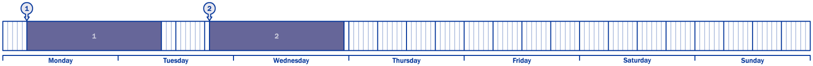

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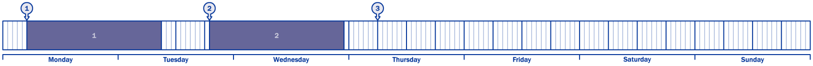

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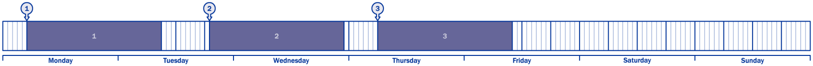

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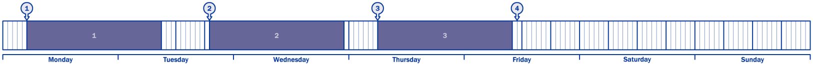

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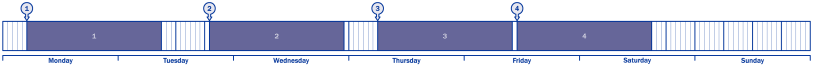

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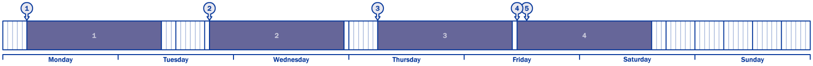

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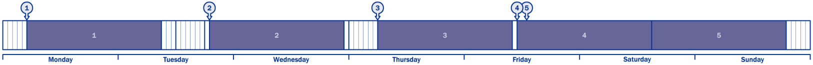


Methodology

In order to assess patient waiting times at the orthopaedic department, we used a variety of queueing models:

- Parametric decomposition approach
  - Kingman equation: closed form
  - Whitt's procedure: algorithm
- Brownian queueing model: heavy traffic setting
- Simulation was used as a validation tool

These models were used to test a variety of scenarios, assessing different levels of impact of service outages (absences and interrupts)

Simulation model

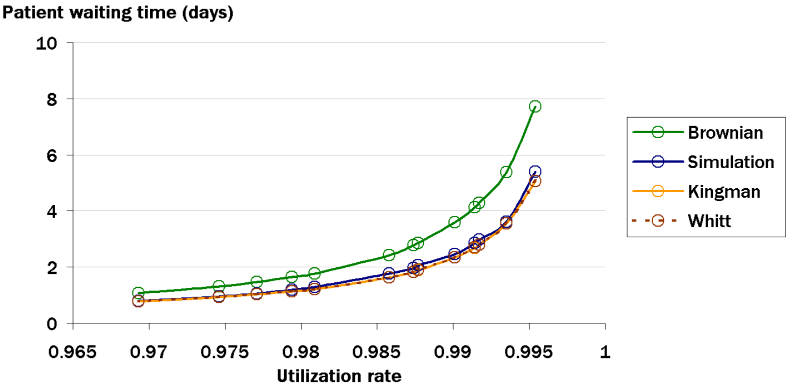
Quick facts:

- 60 modules, 18 classes of patients, different phases of treatment
- Single run simulation for each of the scenarios tested
- Number of patients observed each run: 285,000,000 at surgery, 1,150,000,000 at consultation
- Simulation runtime: 86,000 years
- Resulting statistical precision: standard error < 0.00001

Results: base model

i	1	2	3	4	5
Analytical models					
$\frac{1}{\mu_i}$	0.01257	0.06329	0.79710	5.03237	8.09661
ρ_i	0.99543	0.97854	0.14776	0.75701	0.20396
$C_{s_i}^2$	0.65079	0.60612	14.0786	1.98721	23.4125
$E\left[W_{Kingman}\right]$	5.05894	3.95430	0.79710	5.24027	8.09687
$E\left[W_{Whitt}\right]$	5.05911	3.95298	0.79710	5.20325	8.09664
$E\left[W_{Brownian}\right]$	7.72261	5.41723	0.27924	1.19658	5.00118
i	1	2	3	4	5
Simulation					
$\frac{1}{\mu_i}$	0.01257	0.06329	0.79711	5.03233	8.10131
ρ_i	0.99541	0.97858	0.14775	0.75701	0.20414
$C_{s_i}^2$	0.65796	0.60589	14.0969	1.98918	23.9050
$E\left[W_{Simulation}\right]$	5.40098	3.46204	0.79711	5.11928	8.10131

Results: scenarios



conclusions

- When assessing waiting time at complex hospital systems, parametric decomposition approaches work best
- Hospital decision makers should avoid the high utilization trap
- Decreasing the size, amount and variability of service outages is able to yield significant improvement

Contributions:

- Development of new expressions to model service outages
- Comparison of different modeling techniques

Time for questions