Project Scheduling with Alternative Technologies and Stochastic Activity Durations

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Introduction: Module Networks

Our goal is to maximize the NPV of projects in which:

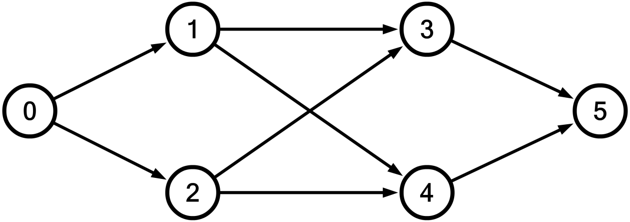
- activities can fail,
- activities that pursue the same result may be grouped in "modules",
- each module needs to be successful for the project to succeed,
- a module is successful if at least one of its activities succeeds

$\Rightarrow$ not all activities in the network have to be started in order for the project to be successful,

$\Rightarrow$ upon failure of all activities in the module, the module fails, resulting in overall project failure.

This is common in R&D (especially in NPD) but also in other sectors: pharmaceuticals, software development, fundraising ...

Example: Definitions



- **(AON) project network with *n* activities**

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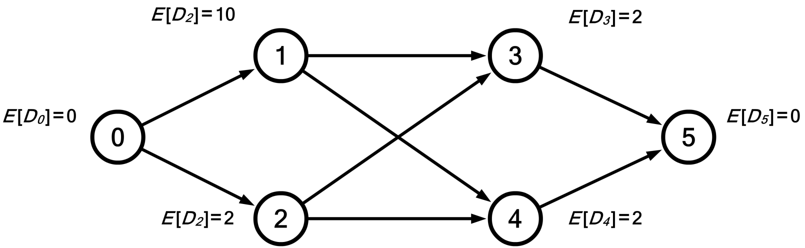

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- **Stochastic activity durations: expected duration $E[D_j]$ of activity j**

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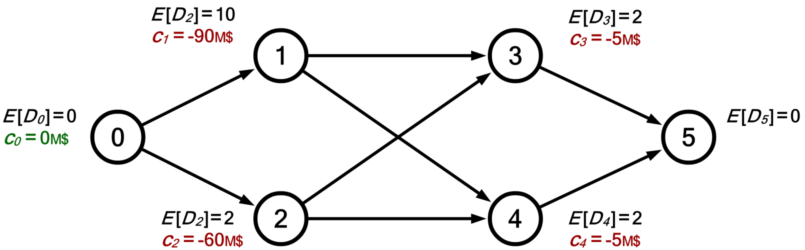

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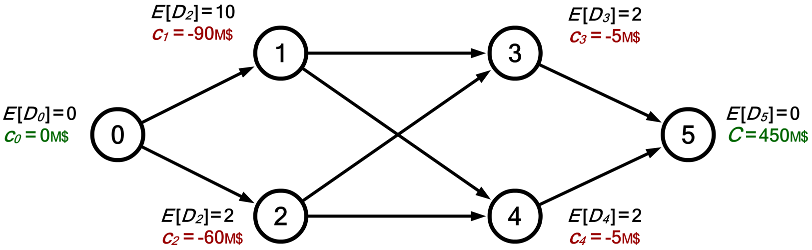

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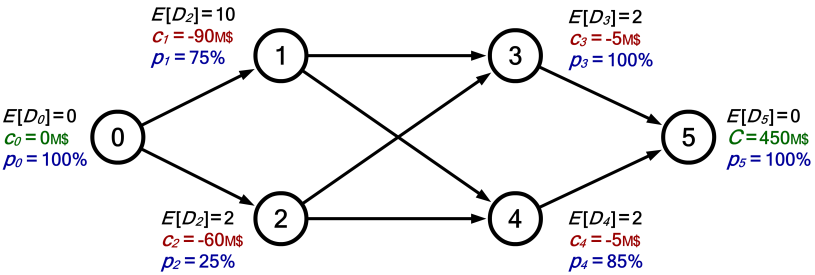

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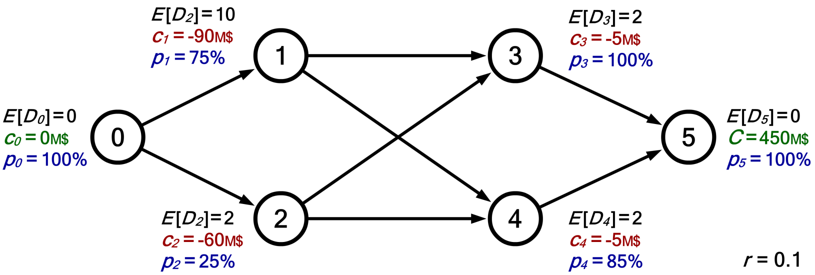

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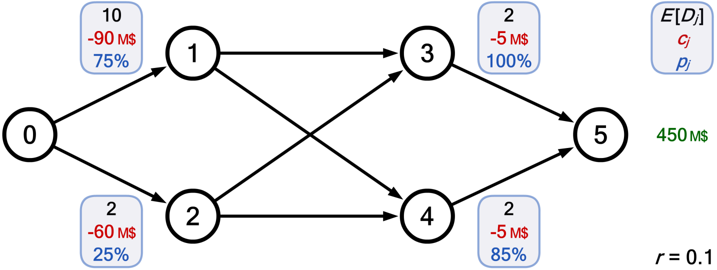

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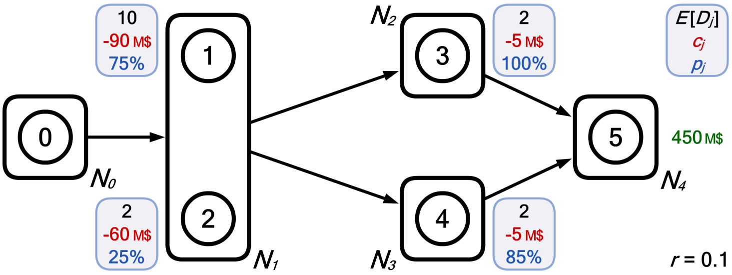

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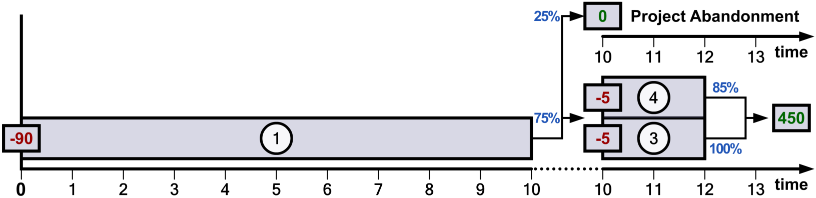
- **Time value of money => discount rate *r***
- ***m* modules N_i**

Example: Policy Π_1

A solution is not a schedule but rather a scheduling policy (even with deterministic durations)

Policy Π_1 results in a NPV of -6.35M$ if activity durations are deterministic



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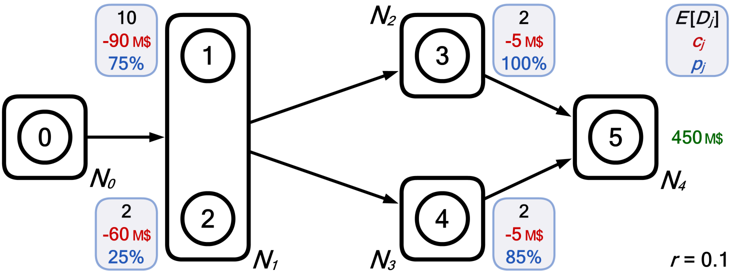

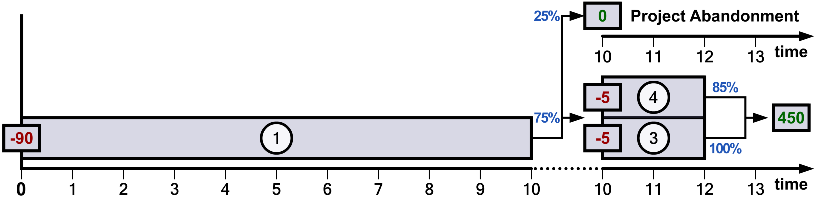

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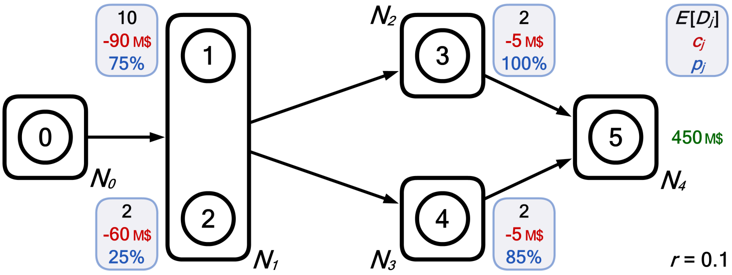

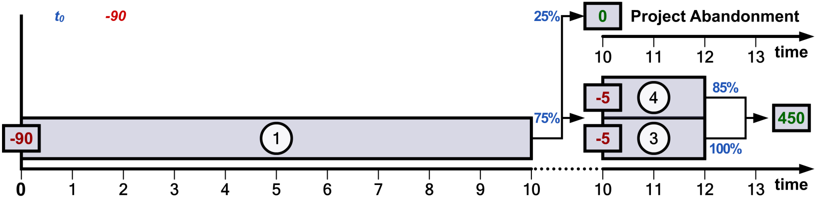

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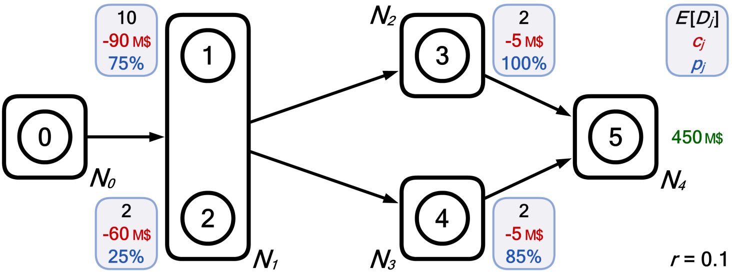

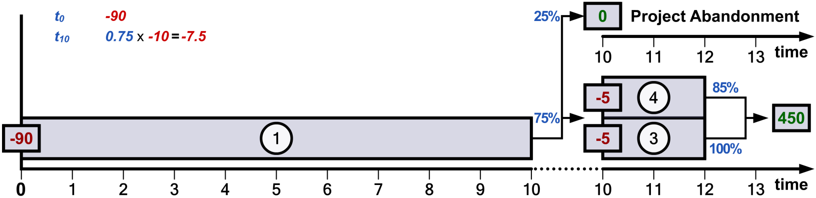

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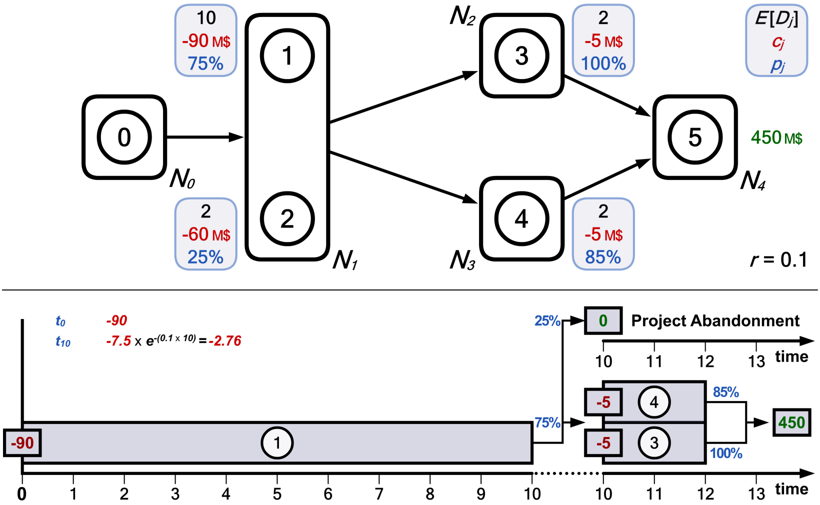

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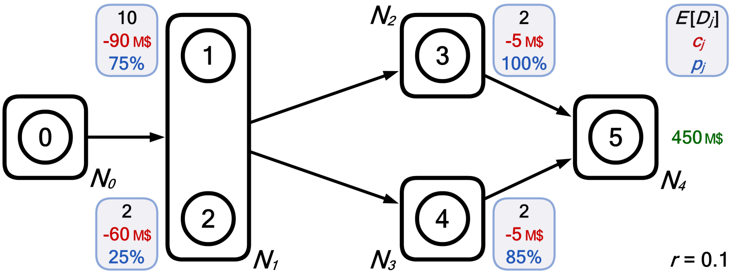

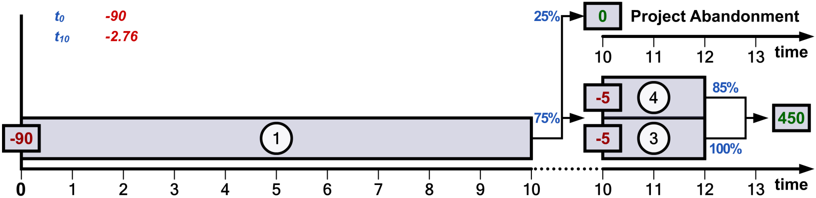

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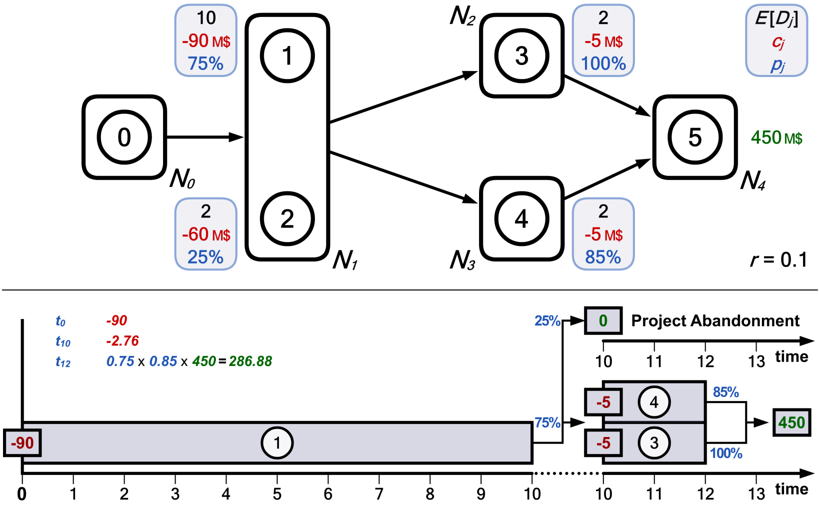

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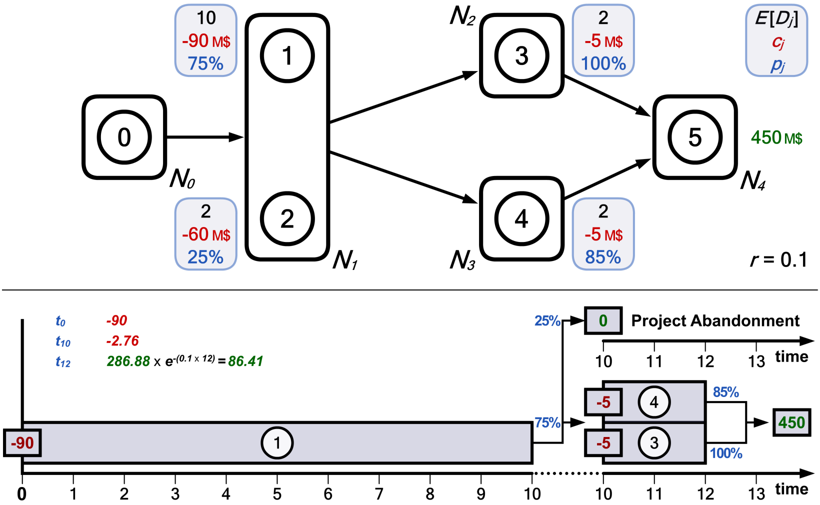

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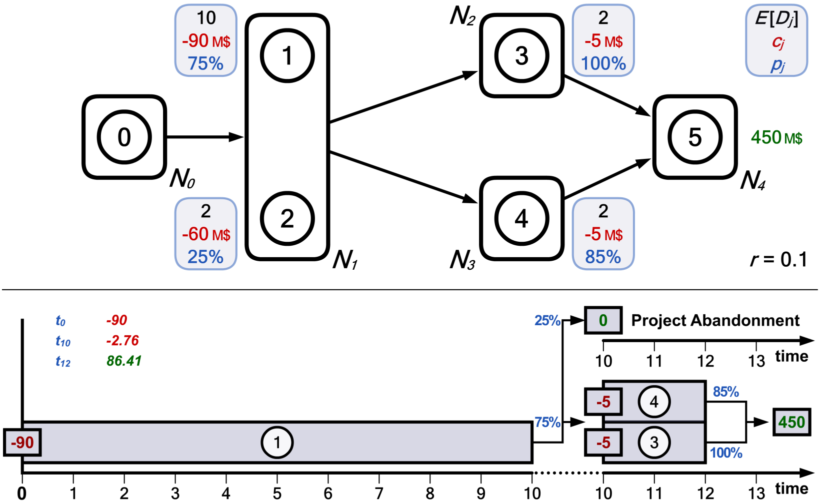

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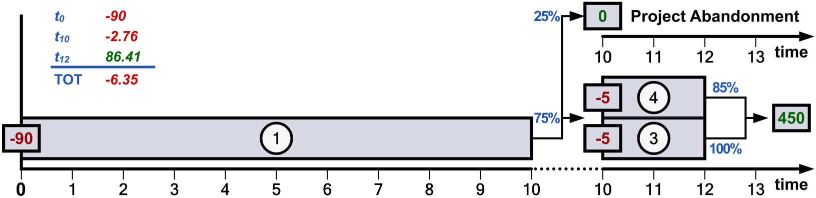


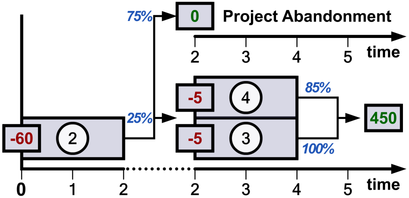
Example: Policy Π_2

A solution is not a schedule but rather a scheduling policy (even with deterministic durations)

Policy Π_1 results in a NPV of -6.35M$ if activity durations are deterministic

Policy Π_2 is optimal for deterministic durations and yields a NPV of 2.05M$

t_0	-60
t_2	-2.04
t_4	64.10
TOT	2.05



Backward SDP-recursion: concepts & definitions

Exponentially distributed activity durations $\Rightarrow$ use of a Continuous-Time Markov Chain (CTMC) to model the statespace.

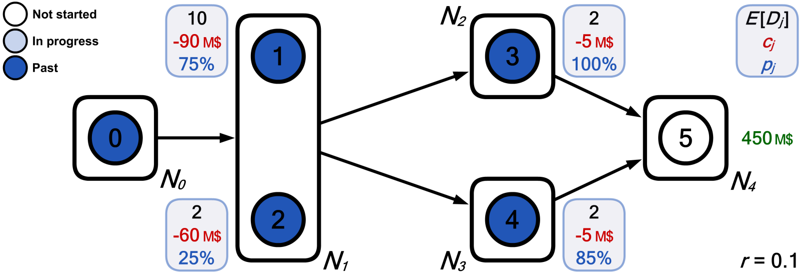
The state of an activity j at time t can be:

- $\Omega_i(t) = 0$: not started,
- $\Omega_i(t) = 1$: in progress,
- $\Omega_i(t) = 2$: past (successfully finished, failed or considered redundant because its module is completed).

The state of the system at a time instance t is given by vector $\mathbf{\Omega}(t) = \{\Omega_0(t), \ldots, \Omega_n(t)\}$.

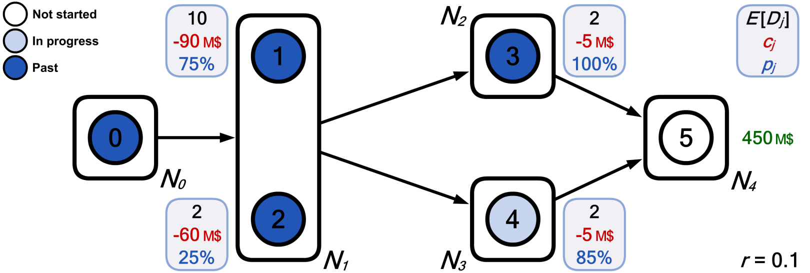
The size of the statespace has upper bound 3^n. Most states do not satisfy precedence constraints $\Rightarrow$ a strict definition of the statespace is required and provided in Creemers et al. (2008).

Example: Stochastic Durations



(2,2,2,2,2,0) [450M$] **Project value upon entry of the final state = project payoff**

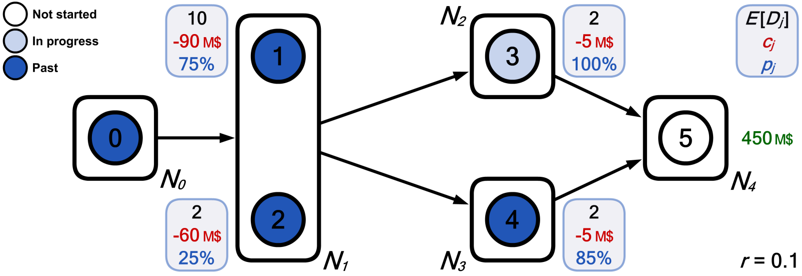
Example: Stochastic Durations



(2,2,2,2,2,0) [450M$]
└→ (2,2,2,2,1,0) [318.75M$]

Discount factor: $(1/D_j)x(r+(1/D_j))^{-1}$
D_4 **= 2 => discount factor = 0.83**
Discounted value upon state entry = 375
p_4 **= 0.85 => NPV upon state entry = 318.75**

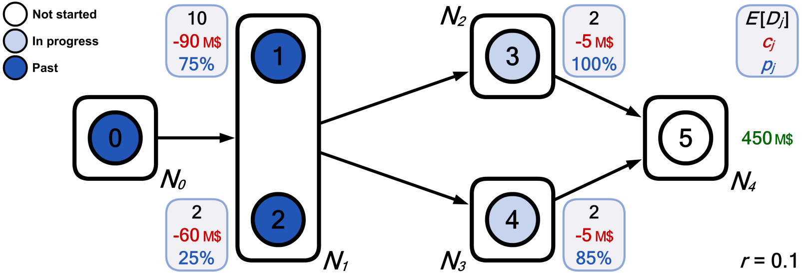
Example: Stochastic Durations



(2,2,2,2,2,0) [450M$]
- → (2,2,2,2,1,0) [318.75M$]
- → (2,2,2,1,2,0) [375M$]

Discount factor: $(1/D_j)x(r+(1/D_j))^{-1}$
D_3 = 2 => discount factor = 0.83
Discounted value upon state entry = 375
p_3 = 1.00 => NPV upon state entry = 375

Example: Stochastic Durations



(2,2,2,2,2,0) [450M$]
→ (2,2,2,2,1,0) [318.75M$]
→ (2,2,2,1,2,0) [375M$]
→ (2,2,2,1,1,0) [289.77M$]

Discount factor = 0.91

Probability of finishing activity *j* first : $(1/D_j)x(SUM(1/D_j))^{-1}$

=> Probability 3 finishes first = 50% & p_3 = 100%

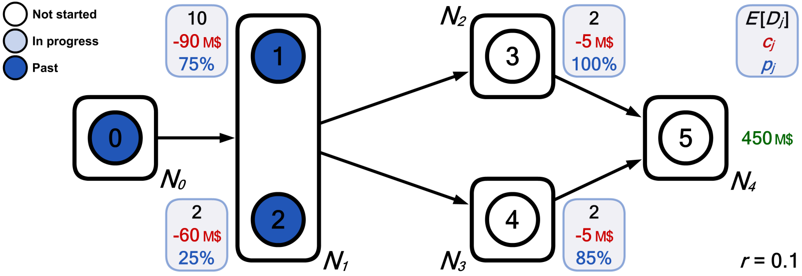
0.5 x 0.91 x 1.00 x 318.75 = 144.89

=> Probability 4 finishes first = 50% & p_4 = 0.85%

0.5 x 0.91 x 0.85 x 375 = 144.89

=> NPV upon state entry = 289.77

Example: Stochastic Durations



(2,2,2,2,2,0) [450M$]
- → (2,2,2,2,1,0) [318.75M$]
- → (2,2,2,1,2,0) [375M$]
  - → (2,2,2,1,1,0) [289.77M$]
    - → (2,2,2,0,0,0) [279.77M$]

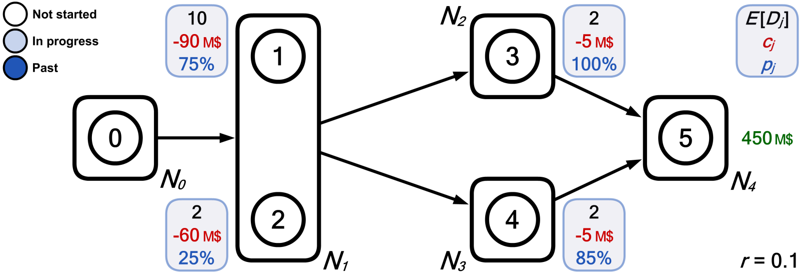
3 possible decisions (pick the optimal one):

- Start activity 3 => incur cost c_3 = -5M$
=> end up in (2,2,1,0,0)

- Start activity 4 => incur cost c_4 = -5M$
=> end up in (2,2,0,1,0)

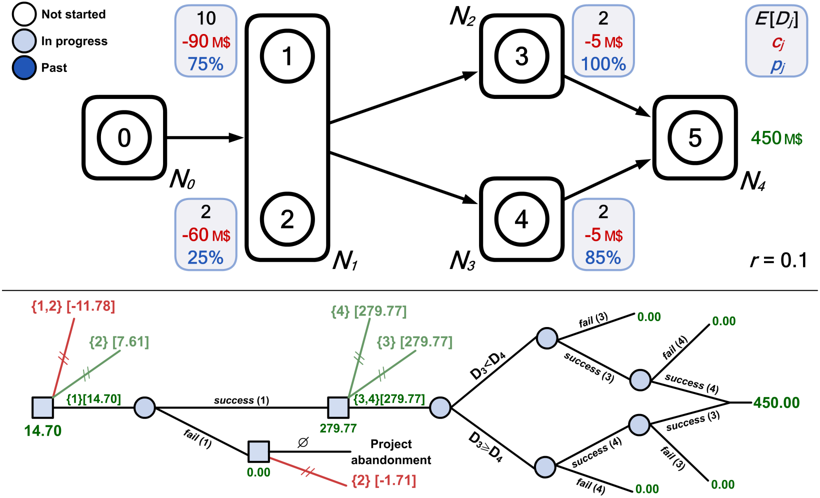
- Start activity 3 & 4 => incur cost $c_3 + c_4$ = -10M$
=> end up in (2,2,1,1,0)[289.77M$]

Example: Stochastic Durations



(2,2,2,2,2,0) [450M$]
→ (2,2,2,2,1,0) [318.75M$]
→ (2,2,2,1,2,0) [375M$]
→ (2,2,2,1,1,0) [289.77M$]
→ (2,2,2,0,0,0) [279.77M$]
→ (...)
→ (0,0,0,0,0,0) [14.70M$]

Example: Stochastic Durations



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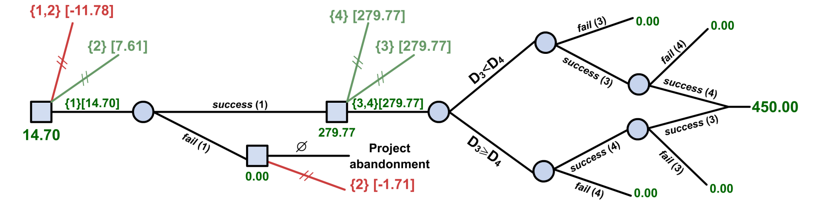
Policy Π_2 is optimal for deterministic durations and yields a NPV of 2.05M$

For stochastic durations, policy Π_1 is optimal with a NPV of 14.70M$



Results & Future Research

Computational results:

- 100 project networks were generated varying in size from 75 activities up to 120 activities. Out of these project networks, 75 have been solved to optimality.
- Computation times vary from less than a second to a maximum of 81,593 seconds. The average computation time for those networks solved amounts to 4,808 seconds.
- The main determinant of the computation time is the density of the network.

Future research:

- Using the model to generate insights in the use of modules
- General activity durations using Phase-Type distributions
- Resources

Time for questions