The Impact of service epochs on waiting times in a healthcare environment

ORAHS'2007

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Problem description

- Problem setting: healthcare and other services
- Measures of interest:
  - Patient waiting time
  - Staff overtime
- Methodology: queueing theory
  - Focus on manufacturing
  - Healthcare modeling requires distinct approach

A basic queueing system

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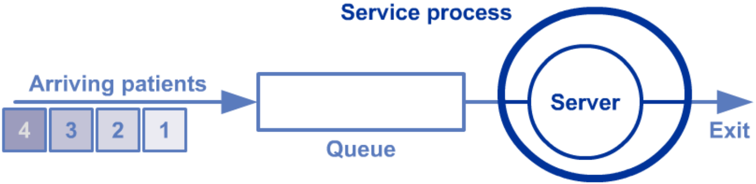

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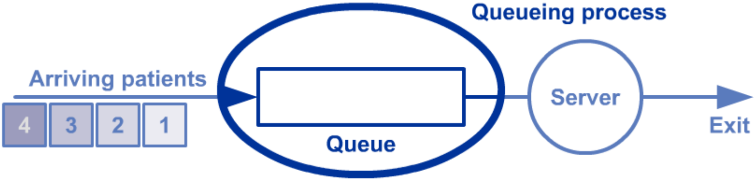

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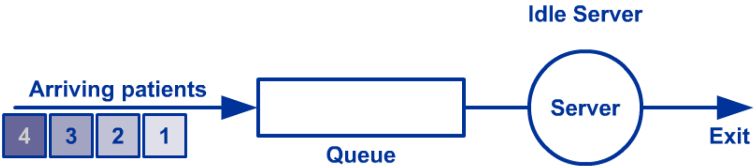

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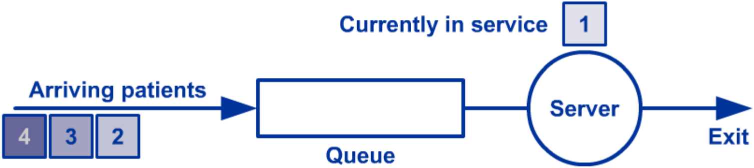

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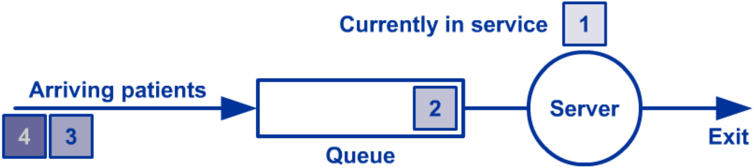

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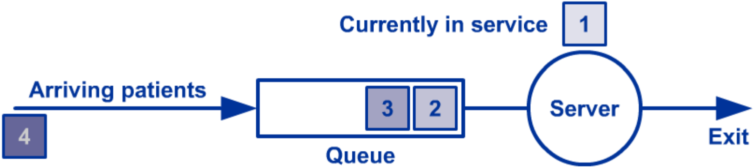

A basic queueing system

Problems in healthcare modeling

- Queue discipline
- Time varying demand
- Waiting creates additional work
- Service outages (absences and interrupts)
- Service epochs

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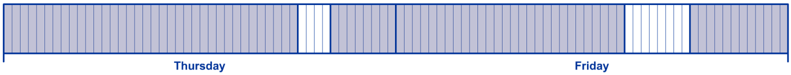

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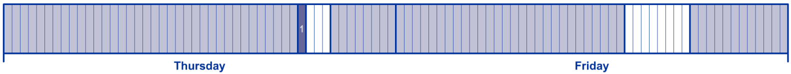

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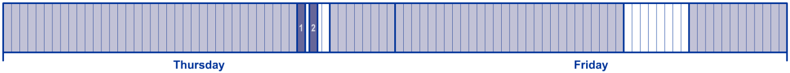

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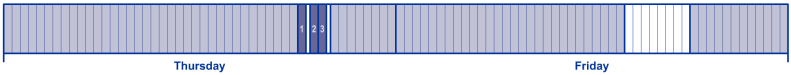

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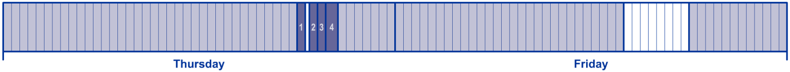

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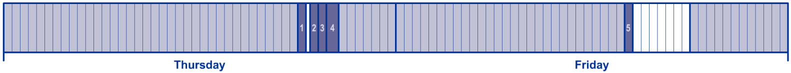

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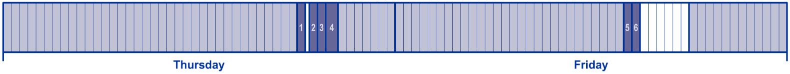

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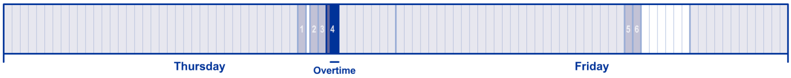

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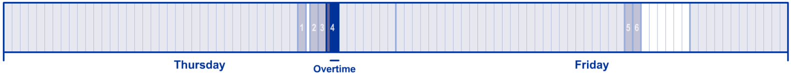

Performance measures of interest:

- Patient waiting time
    - At the waiting list
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- Staff overtime

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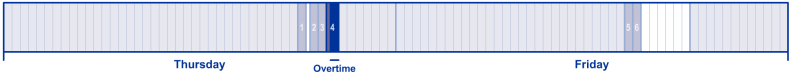

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Two methodologies apply:

- Availability
- Vacation models

Availability

- Rescales the service process in order to fit a predefined uniform time scale (e.g. 24 hours per day, 7 days per week)
- Example: doctor's office with opening hours on Thursday from 6 PM until 8 PM and on Friday from 2 PM until 6 PM
- Availability:

$$A = \frac{6}{168} = \frac{1}{28}$$

Availability

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- Example: doctor's office with opening hours on Thursday from 6 PM until 8 PM and on Fridays from 2 PM until 6 PM
- Mean and variance of the rescaled service times:

$$\frac{1}{\mu} = \frac{1}{A\mu_0}$$

$$\sigma^2 = \frac{\sigma_0^2}{A^2}$$

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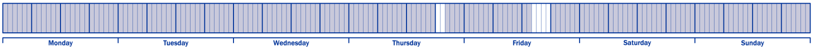

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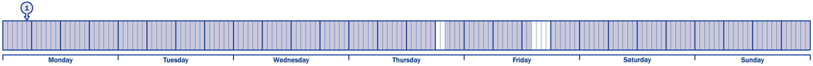

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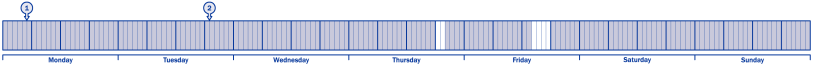

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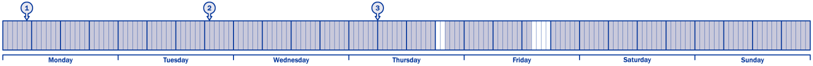

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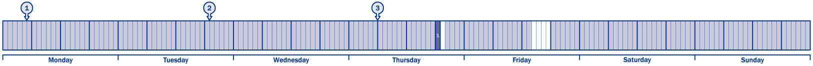

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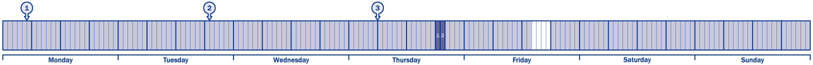

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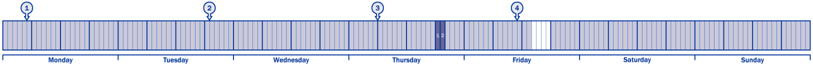

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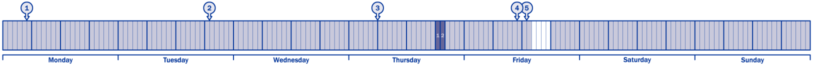

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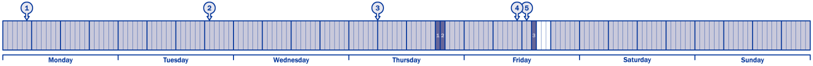

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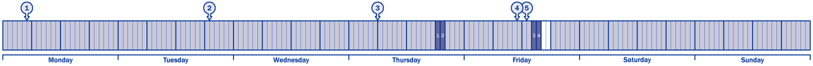

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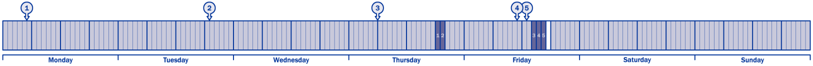

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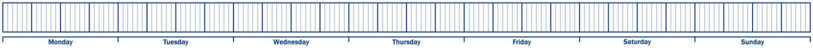

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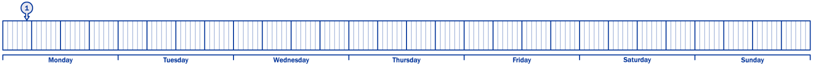

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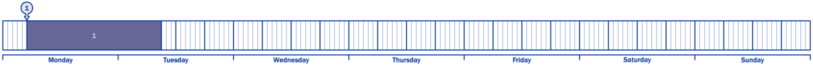

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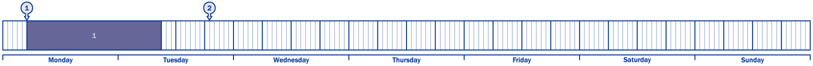

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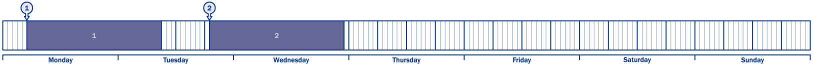

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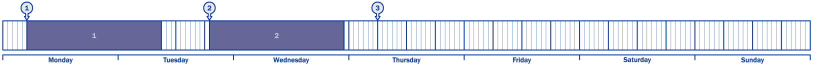

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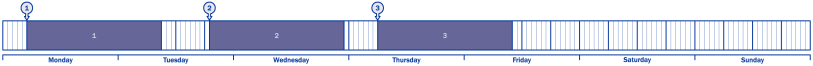

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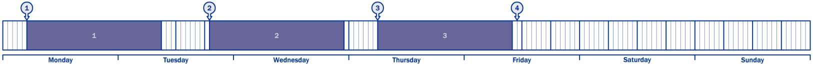

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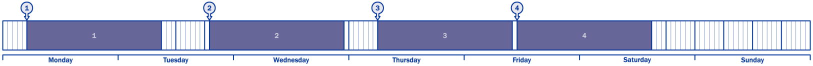

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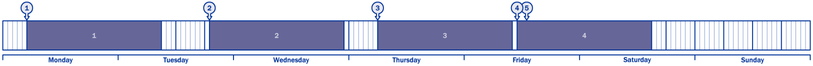

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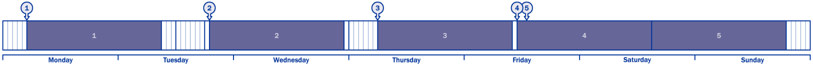

Why not use availability?

- Requirement to know capacity in advance
- Availability is inaccurate at modeling service epochs
- Simulation study:
  - Doctor's office, opening hours on Thursday from 6 PM until 8 PM and on Friday from 2 PM until 6 PM
  - On Thursday a maximum of 4 patients may be treated, on Friday 8 patients are allowed
  - Low variability service, patients always arrive on time, no unscheduled patients, ...
  - Time between the making of two appointments is highly variable

Why not use availability?

- Requirement to know capacity in advance
- Availability is inaccurate at modeling service epochs
- Simulation study:

$$
\begin{array}{lcl}
\hline
A & = & {}^{6}/_{168} \\
\rho & = & 0.6 \\
C_e^2 & = & {}^{1}/_{3} \\
C_a^2 & = & 4 \\
\hline
\end{array}
$$

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- Requirement to know capacity in advance
- Availability is inaccurate at modeling service epochs
- Simulation study:

Availability approach			
$E[W]$	$=$	1.8958	days
Weeks overtime	$=$	$\emptyset$	
Simulation			
$E[W]$	$=$	3.9959	days
Weeks overtime	$=$	46.18%	

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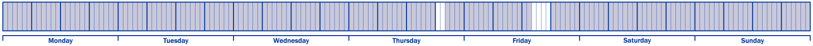

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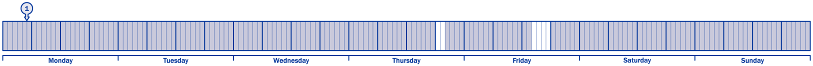

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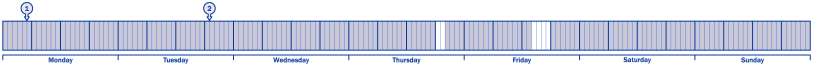

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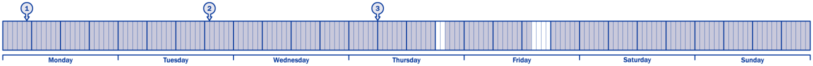

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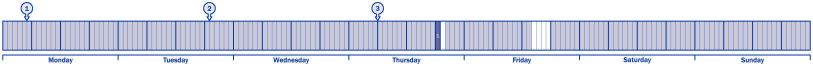

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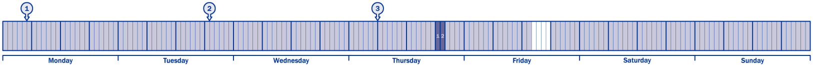

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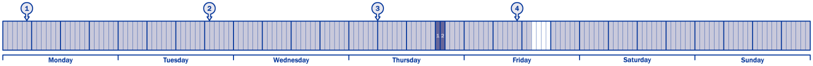

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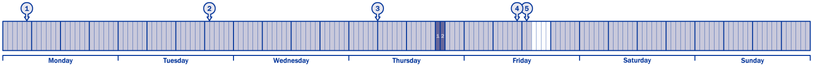

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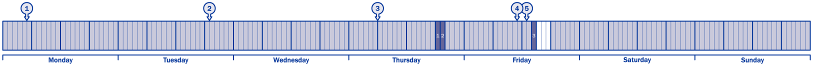

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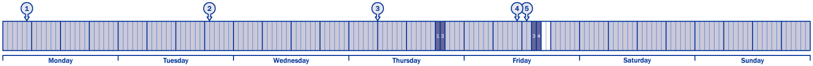

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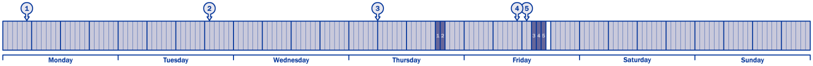

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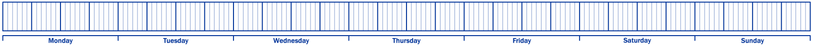

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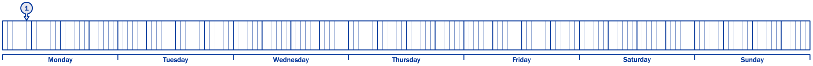

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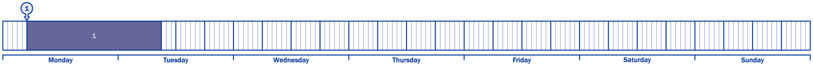

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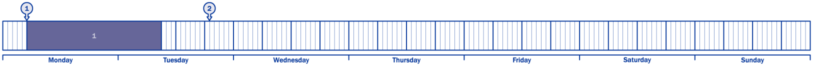

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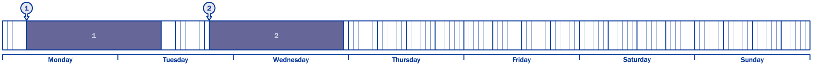

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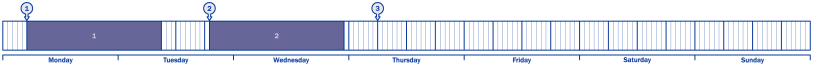

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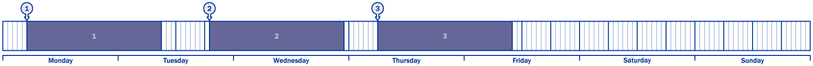

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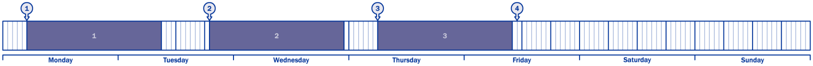

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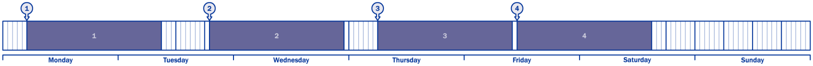

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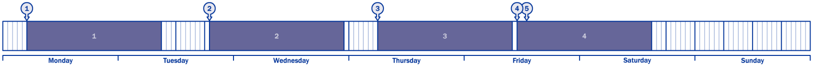

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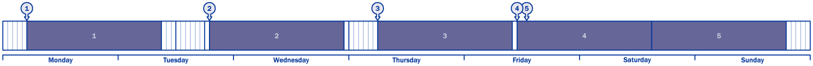

Two queues, two problems

- Two phases in a patients treatment process:
  - External phase (e.g. at home)
  - Internal phase (e.g. at the doctor's office)
- Division of the problem into 2 subproblems:
  - Bulk Transfer Queueing System (BTQS)
  - Service Epoch Markov Chain (SEMC)

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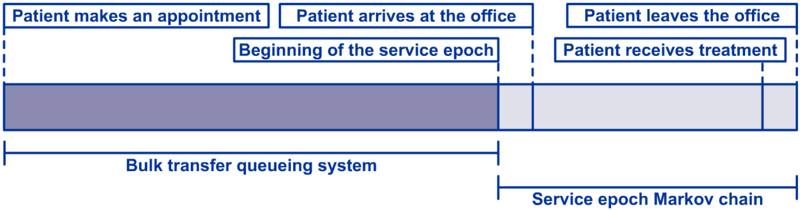

Linking the BTQS and the SEMC

- At the beginning of each service epoch a number of patients is transferred from the BTQS to the SEMC
- The SEMC has two queues:
  - Internal process arrival queue, which holds patients who have yet to arrive
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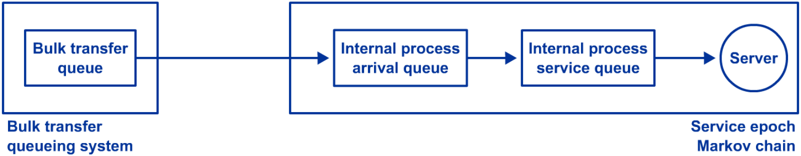

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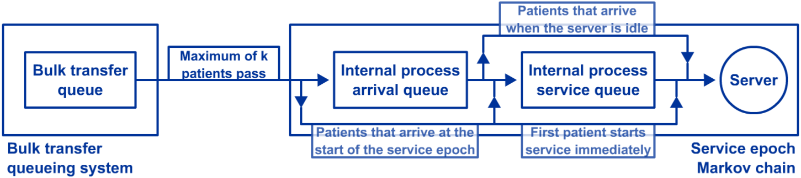

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- The SEMC has two queues:
    - Internal process arrival queue, which holds patients who have yet to arrive
    - Internal process service queue, which represents the waiting room at the doctor's office
- Performance measures of both systems are derived separately and are joined together afterwards

Definition

- The BTQS is a vacation model
  - Gated, k-limited service discipline
  - Bulk service queue with instantaneous service
  - After service patients are transferred towards the SEMC
  - State dependent, deterministic vacations

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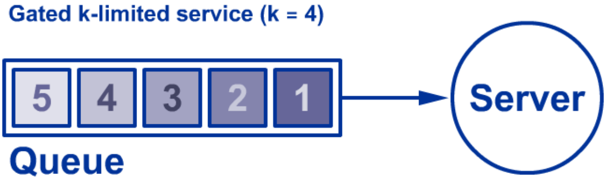

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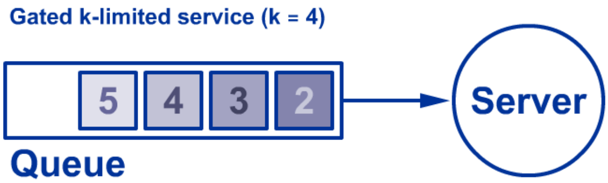

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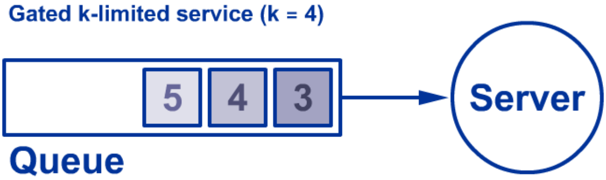

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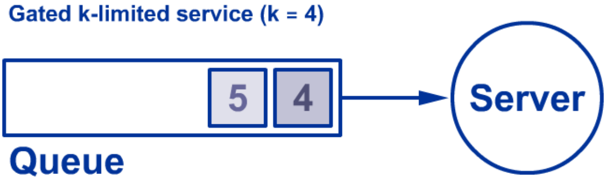

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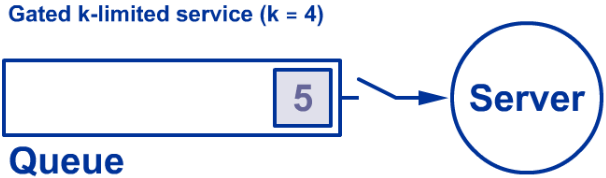

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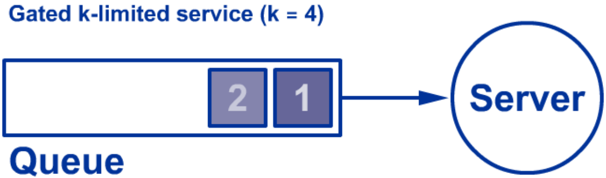

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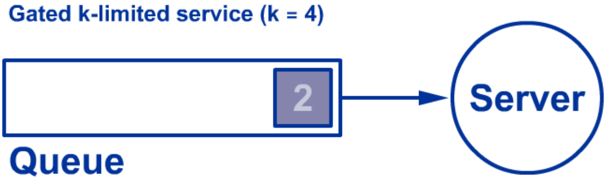

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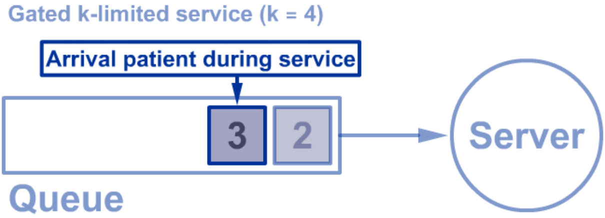

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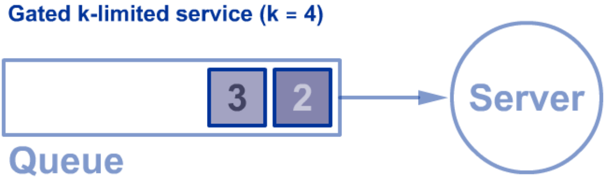

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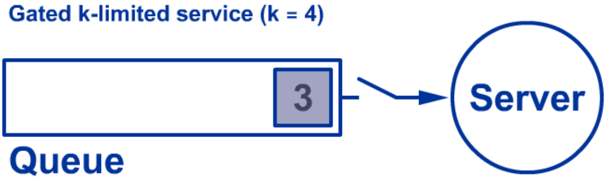

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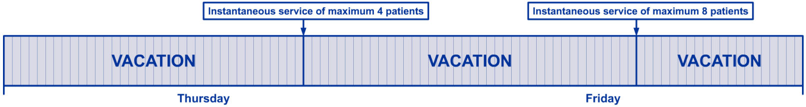

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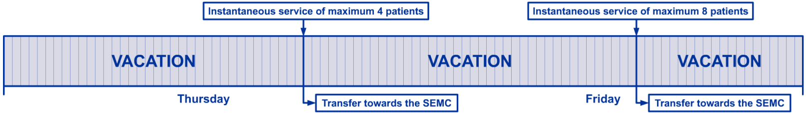

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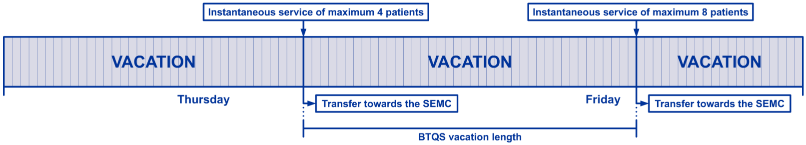

Example

- One service epoch (e.g. Thursday from 6 PM until 8 PM)
- Maximum of 4 patients is allowed
- Arrival rate λ, vacation rate μ

i/j	0	1	2	3	4	5	6	7	...
0	μ	λ	0	0	0	0	0	0	...
1	μ	0	λ	0	0	0	0	0	...
2	μ	0	0	λ	0	0	0	0	...
3	μ	0	0	0	λ	0	0	0	...
4	μ	0	0	0	0	λ	0	0	...
5	0	μ	0	0	0	0	λ	0	...
6	0	0	μ	0	0	0	0	λ	...
7	0	0	0	μ	0	0	0	0	...
...	...	...	...	...	...	...	...	...	...

BTQS: underlying Markov chain

Output

From the analysis of the BTQS we obtain:

- The stationary distribution of the number of patients in queue
- The waiting time of a patient at the BTQS (i.e. part of the waiting time spent in the waiting list)
- The probability of a certain number of patients being transferred towards the SEMC at the beginning of a particular service epoch (i.e. the input of the SEMC system)

Definition

The SEMC is an absorbing Markov chain in which:

- Each state is represented by a triplet $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ where:
    - $\mathcal{A}$ denotes the number of patients in the internal process arrival queue
    - $\mathcal{B}$ denotes the number of patients in the internal process service queue
    - $\mathcal{C}$ denotes the number of patients currently in service
- The absorption time indicates the end of service at a service epoch

Example

- One service epoch on Thursday (from 6 PM until 8 PM)
- 4 patients made an appointment
- none of the patients are present at the doctor's office upon opening
- Arrival rate at the doctor's office λ, service rate μ

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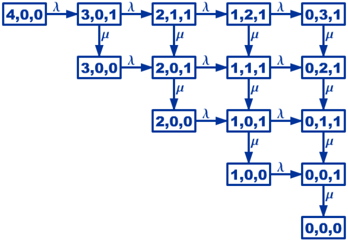
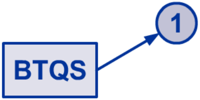


Output

- From the analysis of the SEMC we obtain:
    - The overtime performed at a particular service epoch, given a number of patients transferred from the BTQS
    - The waiting time at both the internal process arrival and service queue at a particular service epoch, given a number of patients transferred from the BTQS
- For each service epoch and each possible number of patients transferred, we need to analyze the SEMC
- Combined with the performance measures of the BTQS, general performance measures may be obtained

Combining both subproblems



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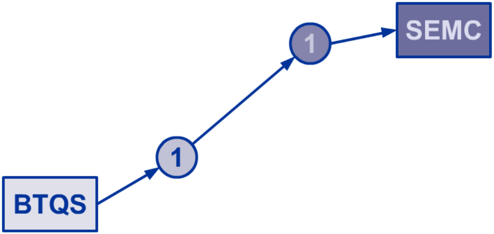

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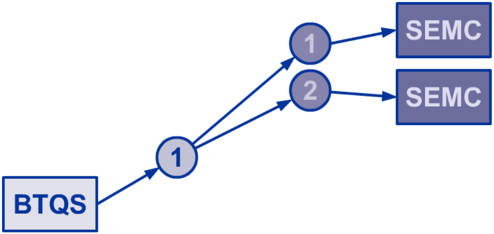

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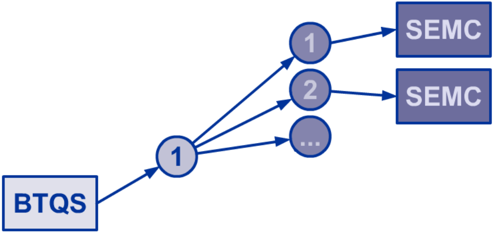

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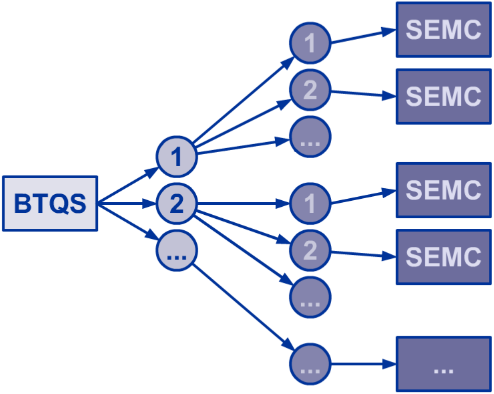

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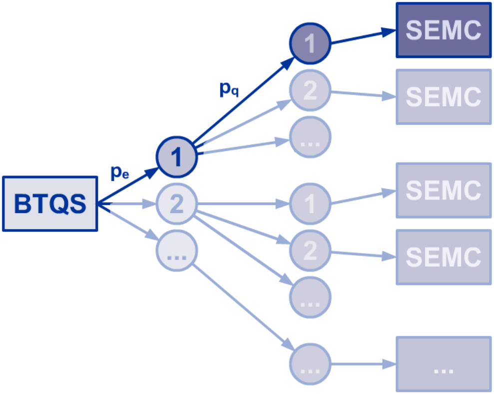

Assumptions

- Use of exponential distribution:
  - Vacation length
  - Service times
  - Interarrival times at the doctor's office
  - Interarrival times between appointments
- Patients:
  - Are assumed to make an appointment (i.e. no unscheduled patients show up)
  - Are assigned the first time slot available
  - Are assumed to arrive and to be served during the assigned service epoch

Numerical example

- Setting: doctor's office with opening hours on Thursday (6 PM until 8 PM; a maximum of 4 patients may be treated)
- Assumptions:
  - Vacation lengths of exponential duration (168 hours)
  - Exponential service time with mean 30 minutes
  - Exponential interarrival time at the waiting list with mean 3,000 minutes
  - Exponential interarrival time at the doctor's office with mean 18 minutes

Numerical example

Parameter		Exact	Simulation
$E[W_\alpha]$	$=$	$40,768$ minutes	$40,759$ minutes
$E[W_\beta]$	$=$	38.625 minutes	38.627 minutes
$E[W_\gamma]$	$=$	20.999 minutes	21.003 minutes
$E[W]$	$=$	$40,828$ minutes	$40,818$ minutes
$E[O]$	$=$	23.894 minutes	23.908 minutes

Conclusions

Contributions:

- Modeling technique that enables the assessment of:
  - Staff overtime
  - Patient waiting time at the waiting list
  - Patient waiting time at the internal facility

Current limitations:

- Use of exponential distribution for vacation lengths and interarrival and service times
- More efficient computation of performance measures is possible
- Various extensions should allow for more realistic models (e.g. unscheduled patients, multiple doctors, . . .)

Upcoming research

- Use of Phase Type distributions to obtain:
  - More realistic models; Phase Type distributions can be used to model a wide variety of existing distributions
  - More detailed performance measures (i.e. not limited to expected values)
- Use of Matrix analytical techniques to optimize computations
- Provide various extensions to the model (unscheduled patients, multiple doctors, . . .)
- Assess impact of size and location of service epochs on performance measures

Time for questions