KATHOLIEKE UNIVERSITEIT LEUVEN

Project Scheduling with Alternative Technologies and Stochastic Activity Durations

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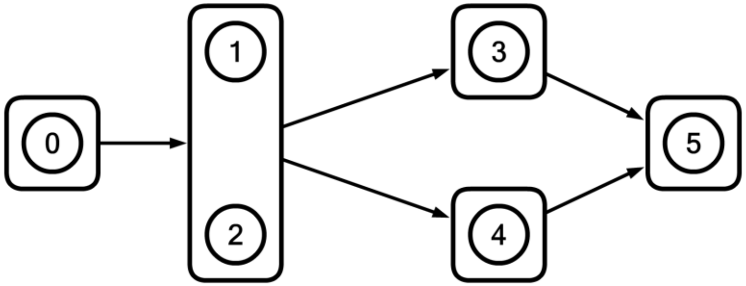
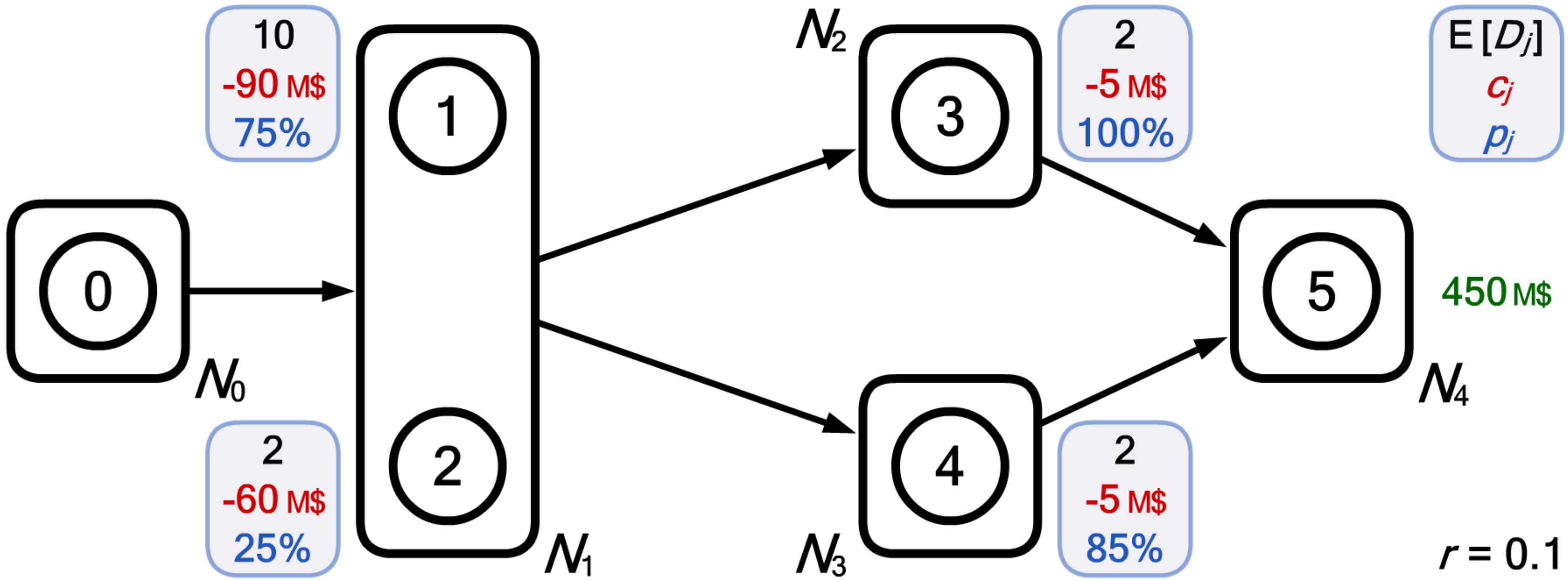
Introduction

- Goal = maximize NPV of projects in which:
  - Activities can fail
  - Activities that pursue the same result may be grouped in "modules"
  - Each module needs to be successful for the project to succeed
  - A module is successful if at least one of its activities succeeds



- This is common in R&D (especially in NPD) but also in other sectors: pharmaceuticals, software development, …

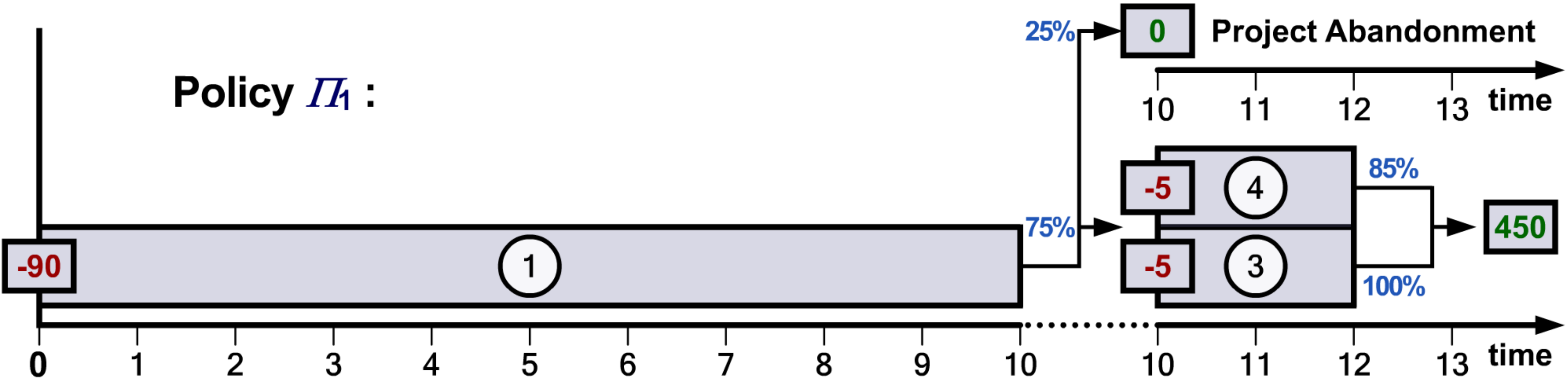
Definitions



- Project network with *n* activities (activity = on the node)
- Stochastic activity durations: expected duration $E[D_j]$ of activity *j*
- Expected-NPV objective: cash flow c_j is incurred at the start of activity *j*
- End-of-project payoff *C* obtained upon overall project success
- Failures: each activity *j* has a probability of technical success p_j
- Time value of money => discount rate *r*
- *m* modules N_i

Deterministic durations

A solution is not a schedule but rather a scheduling policy (even with deterministic durations)



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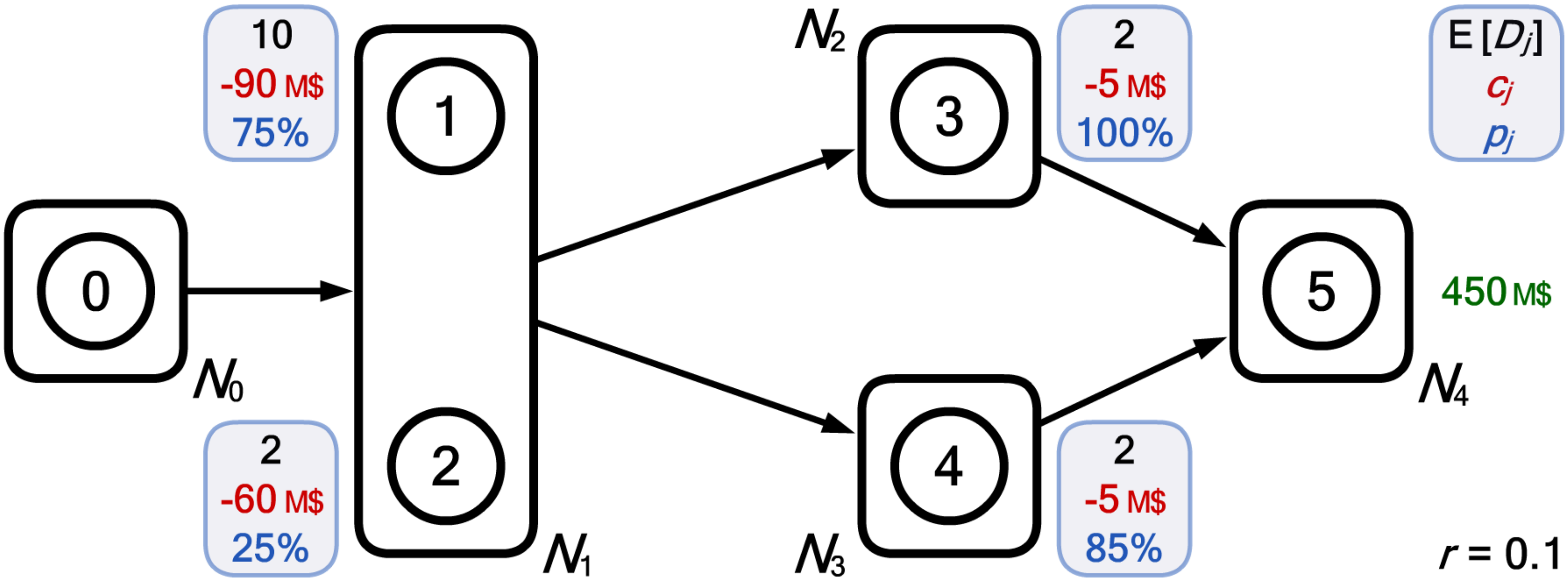

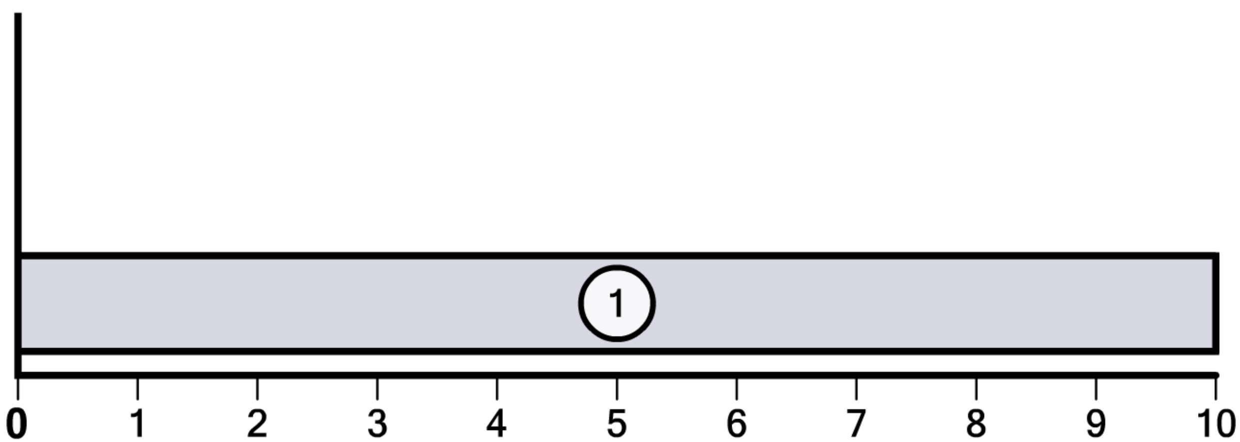

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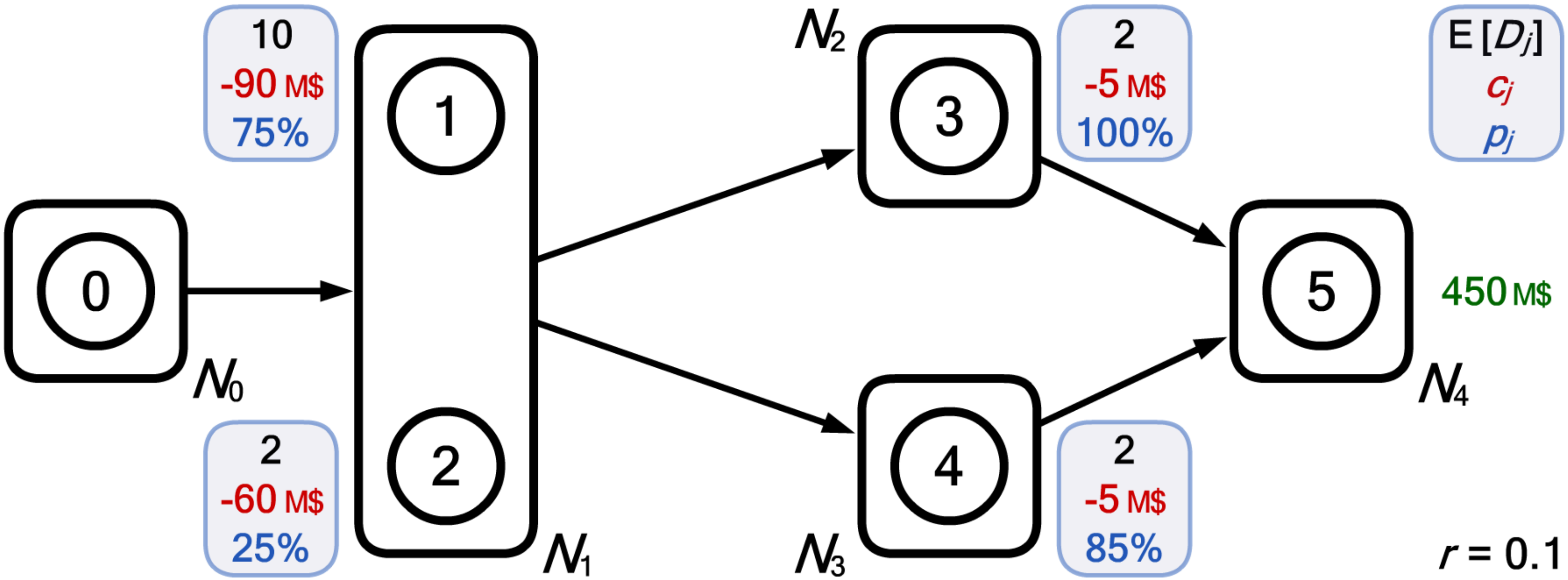

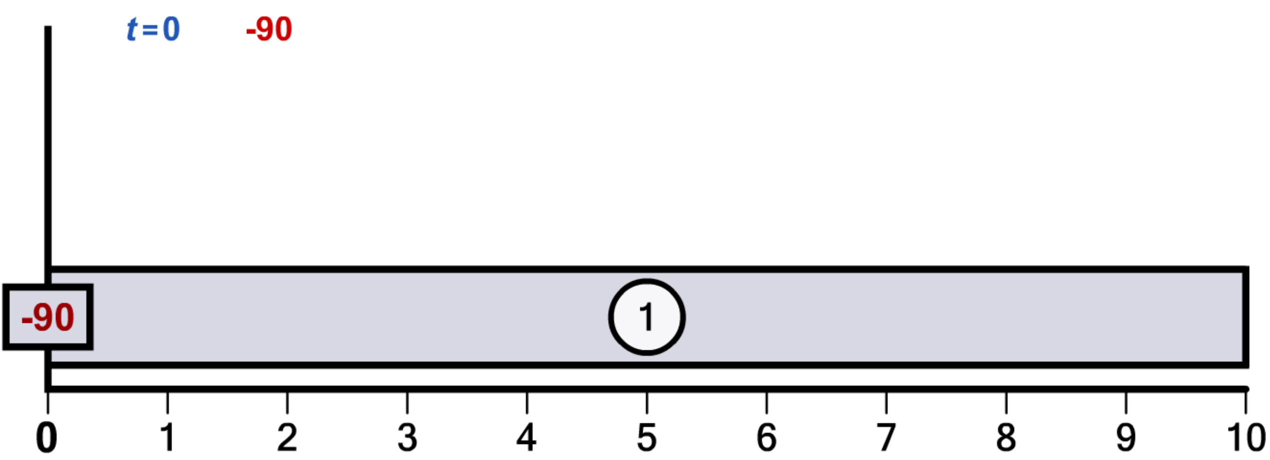

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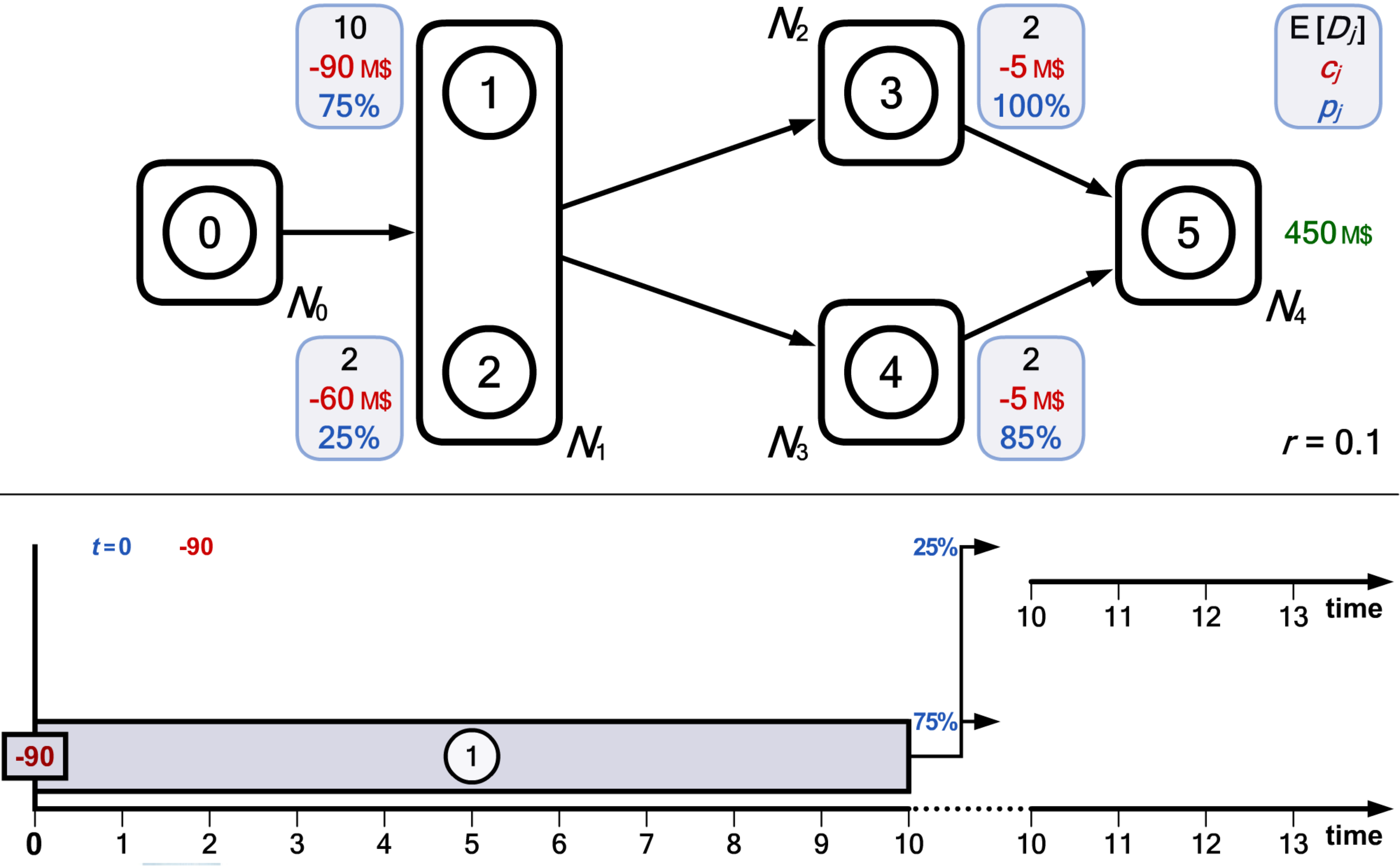

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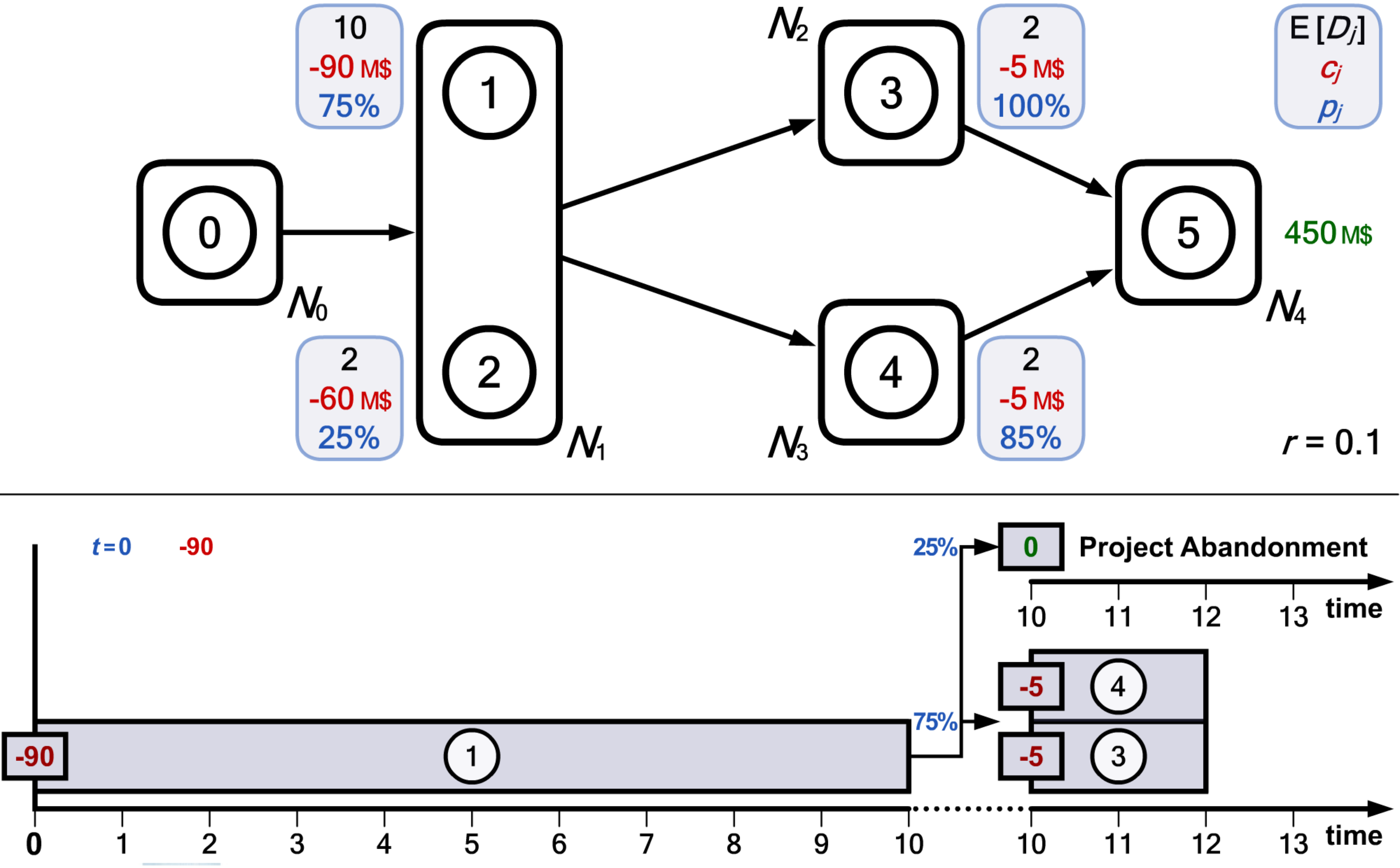

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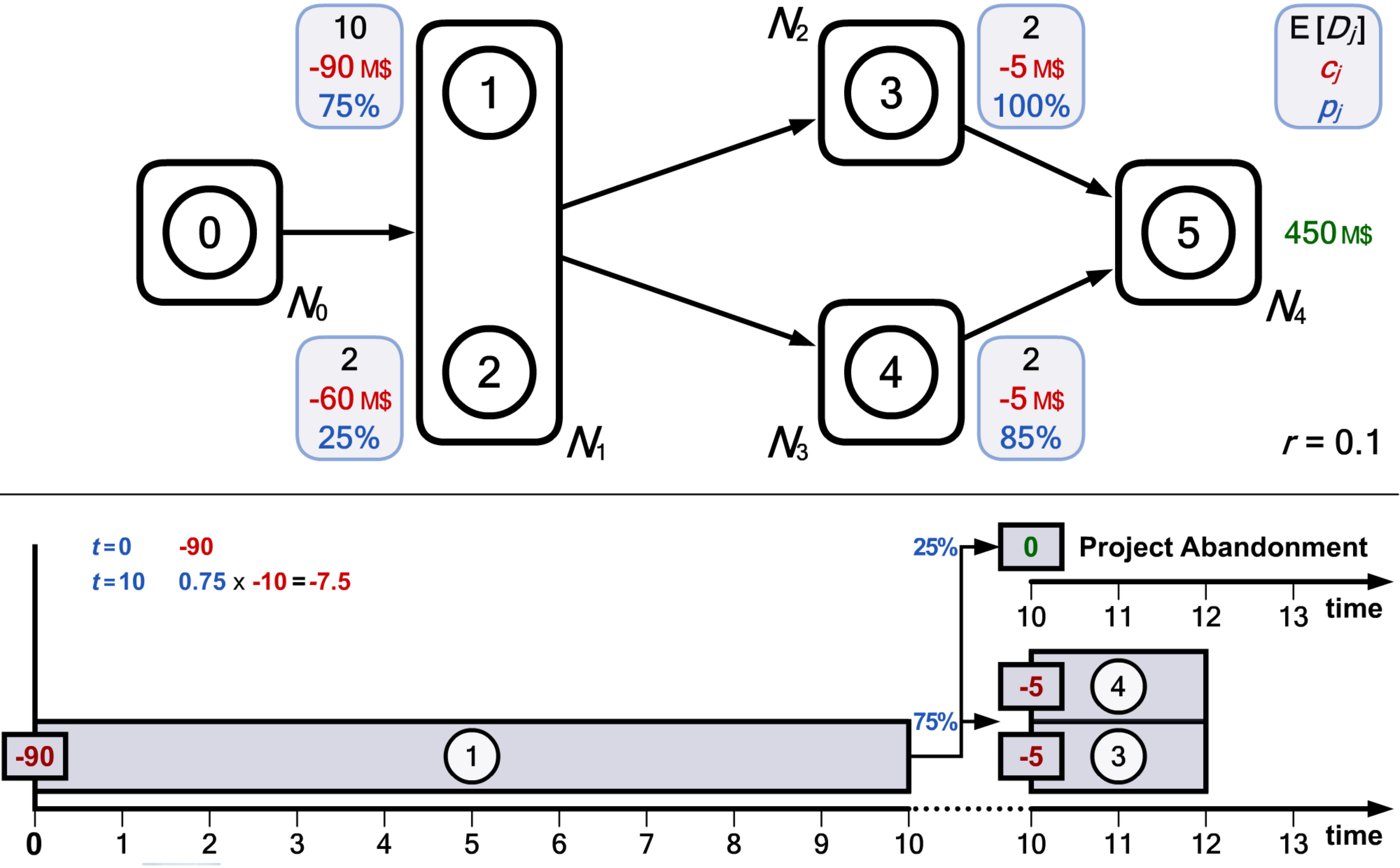

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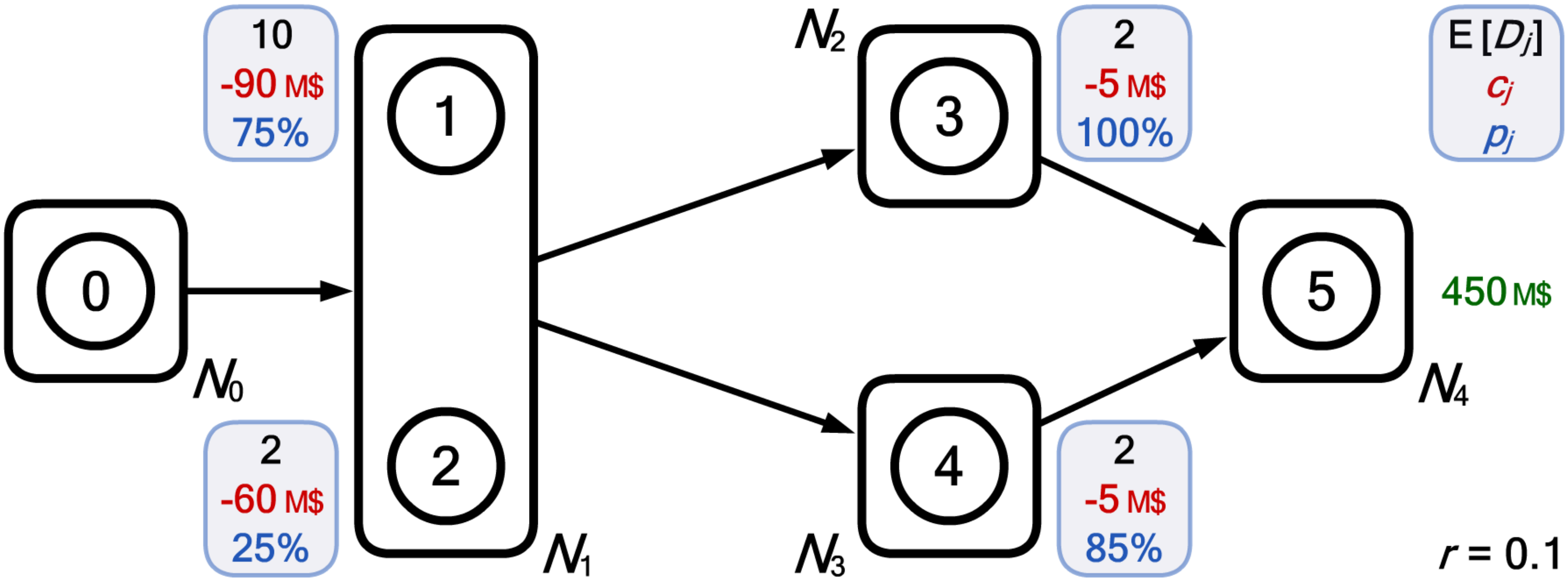

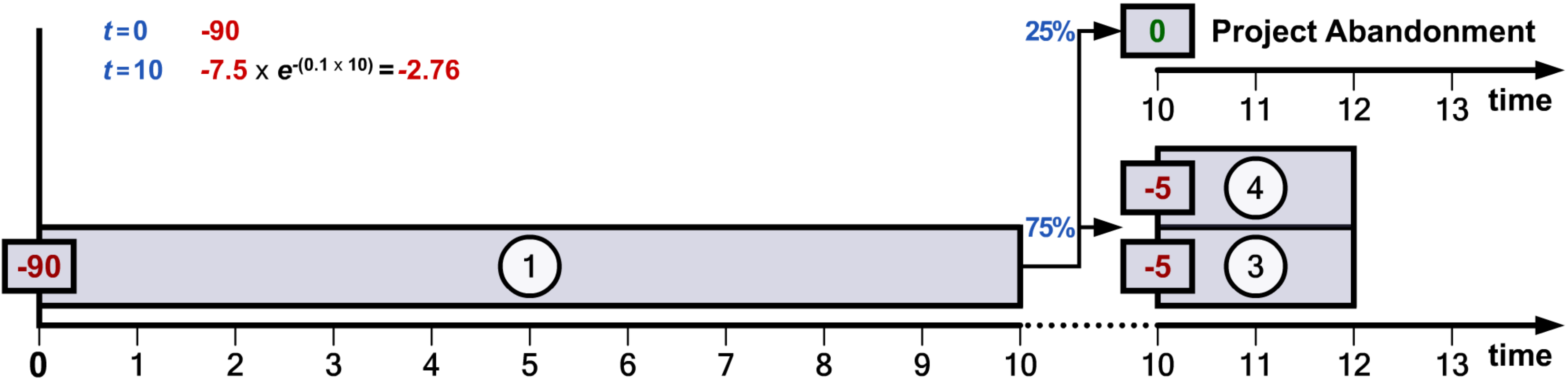

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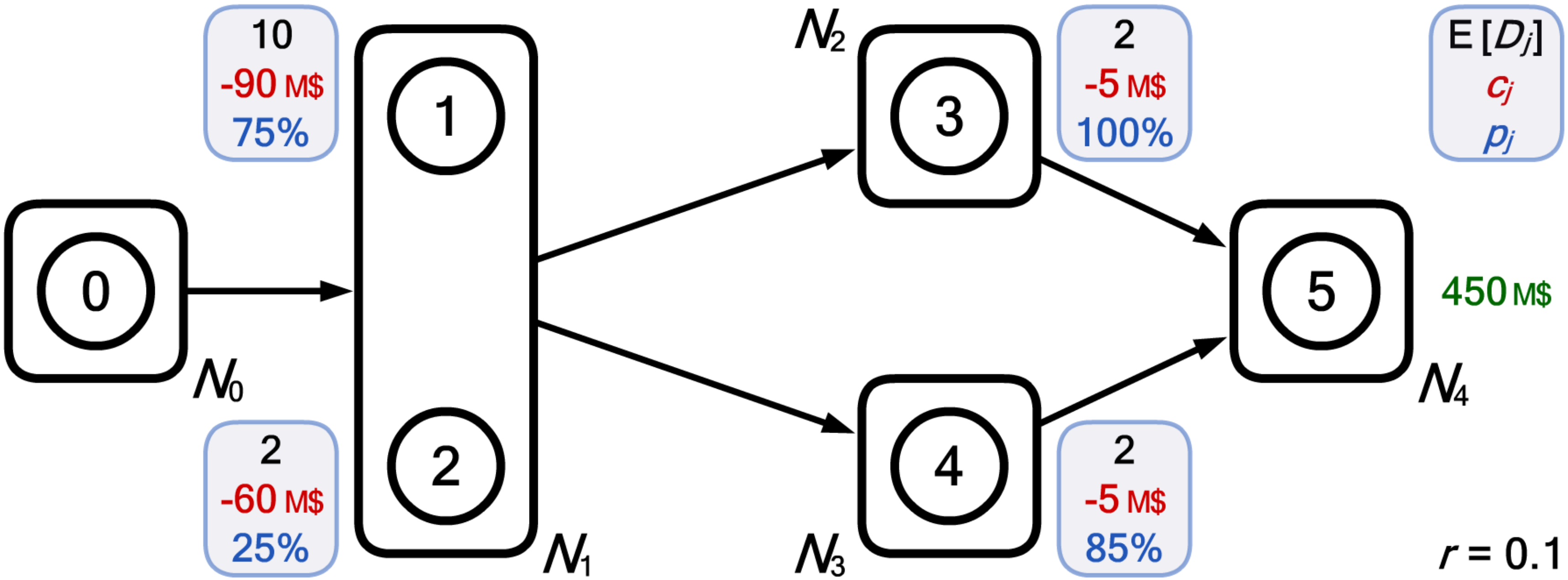

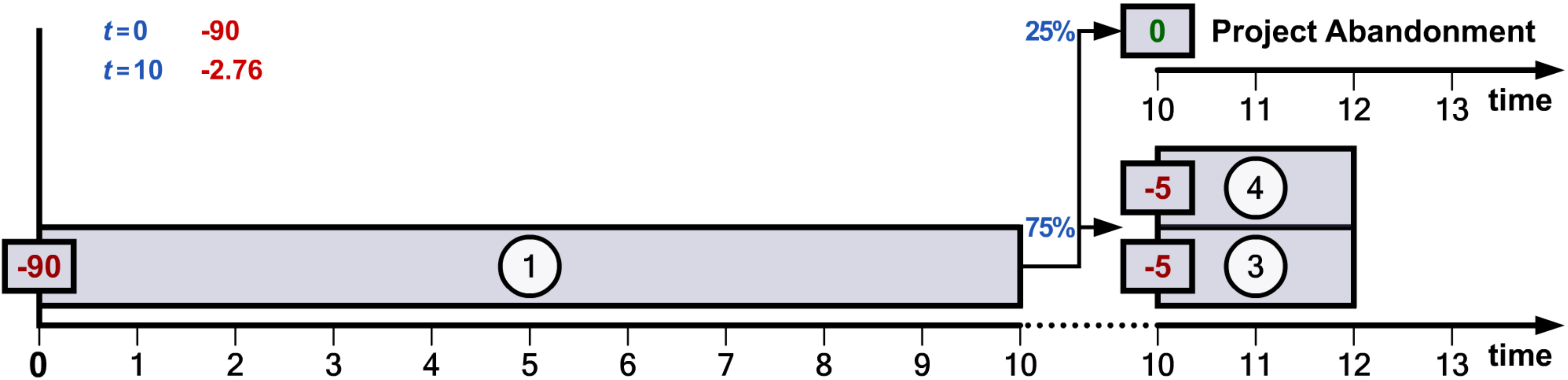

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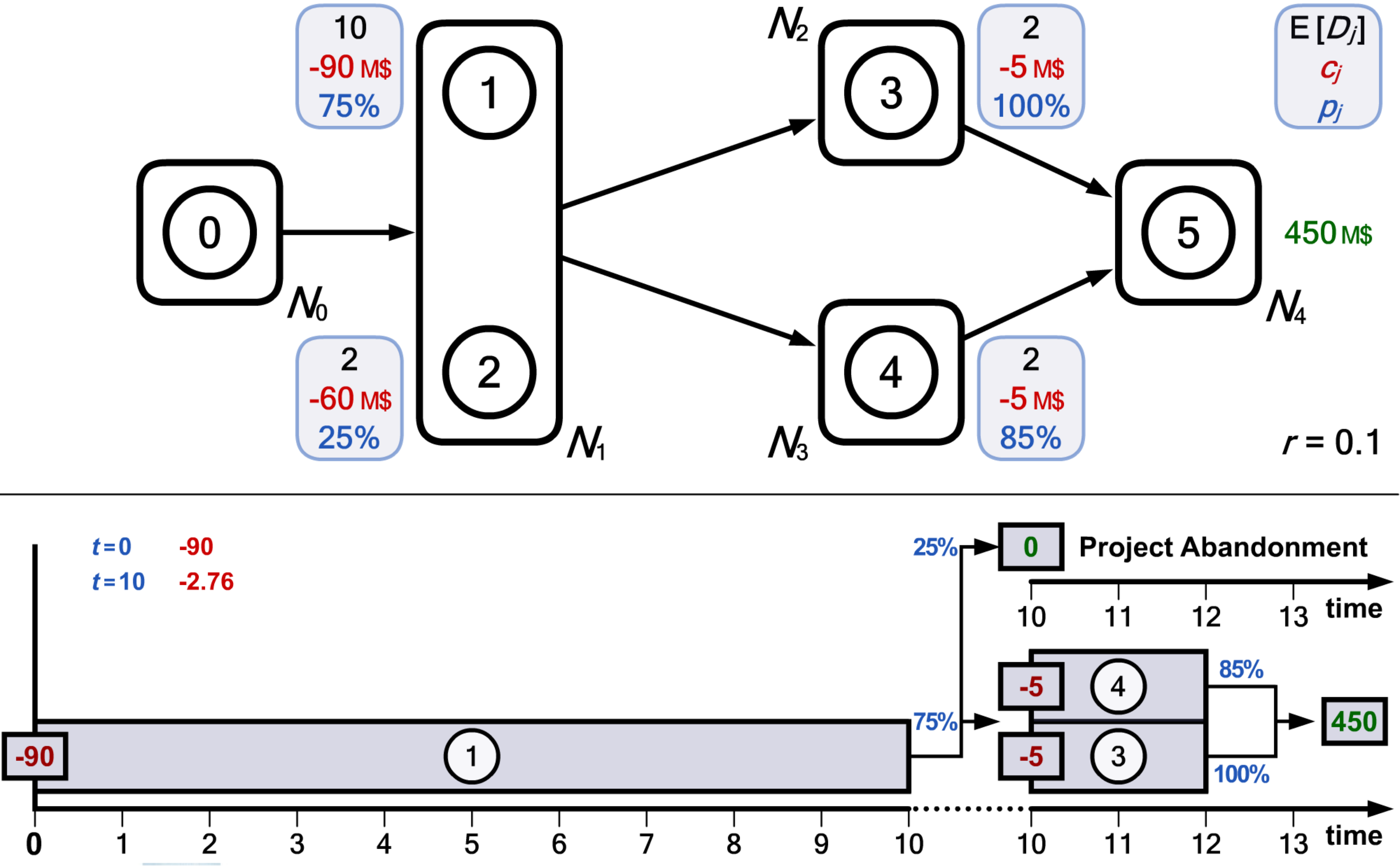

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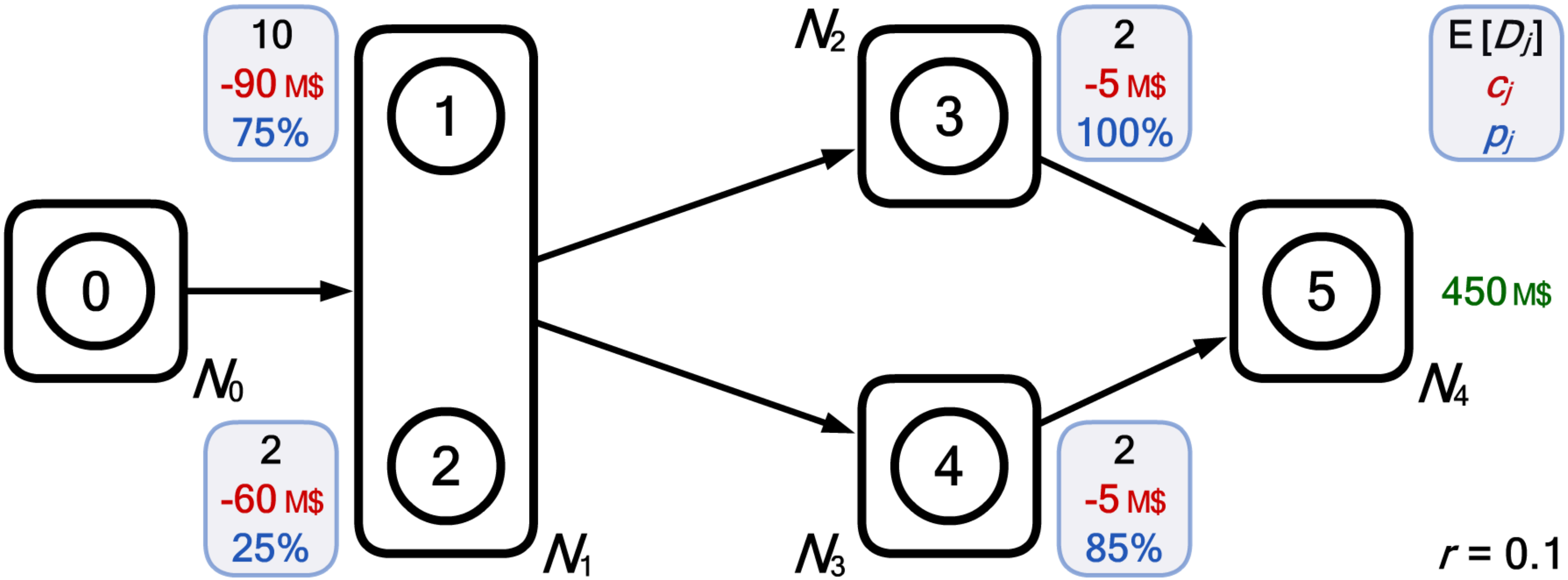

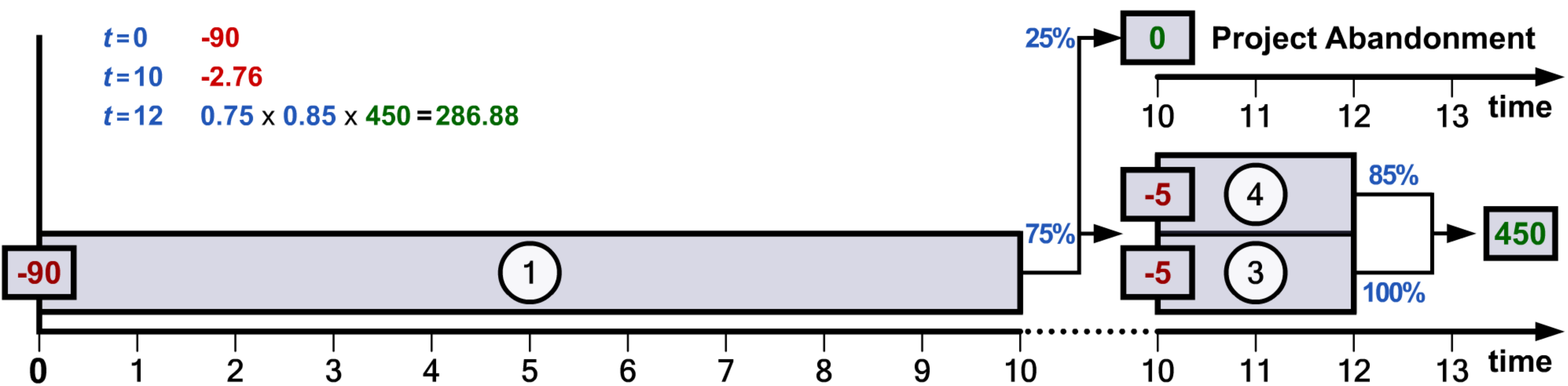

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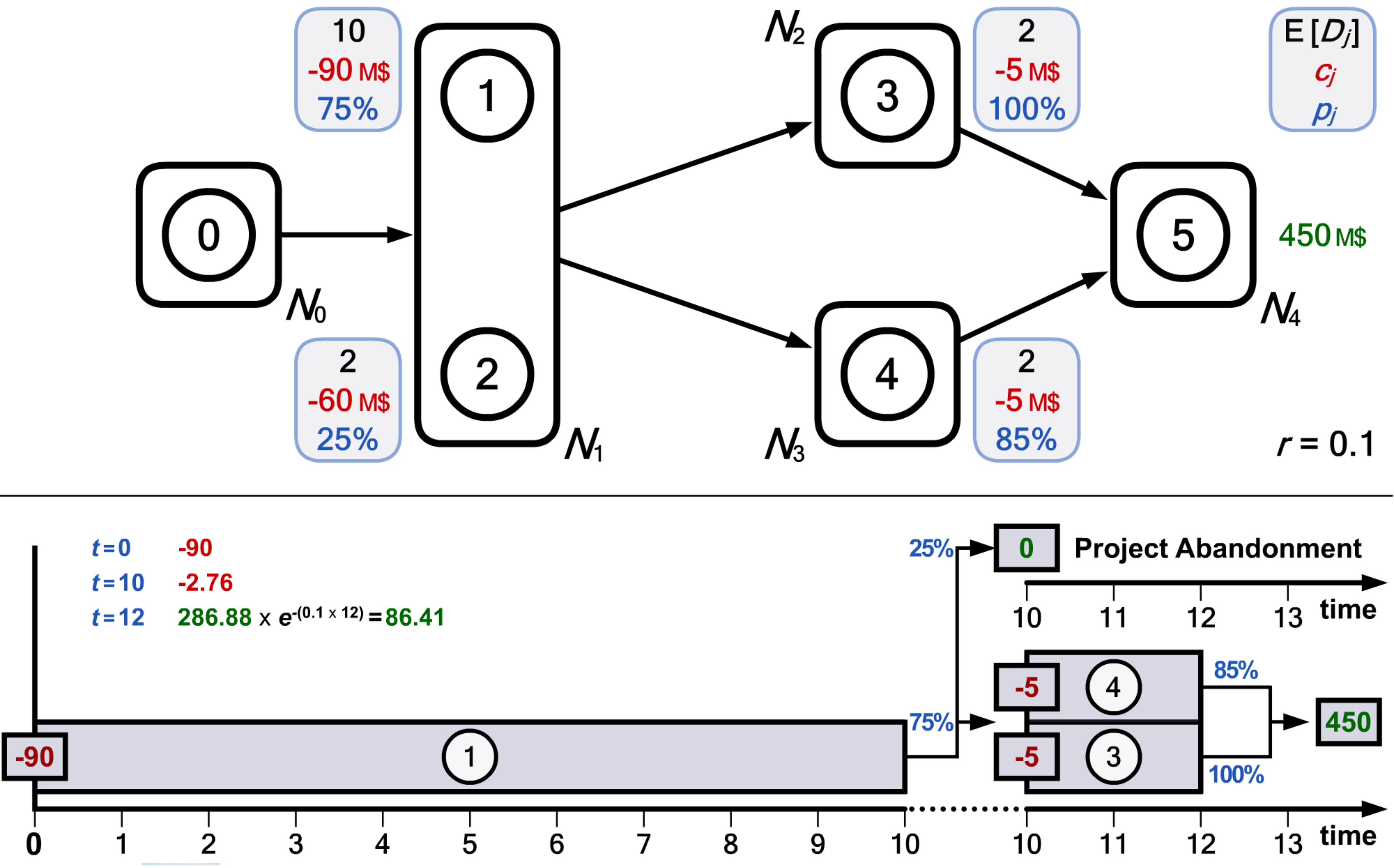

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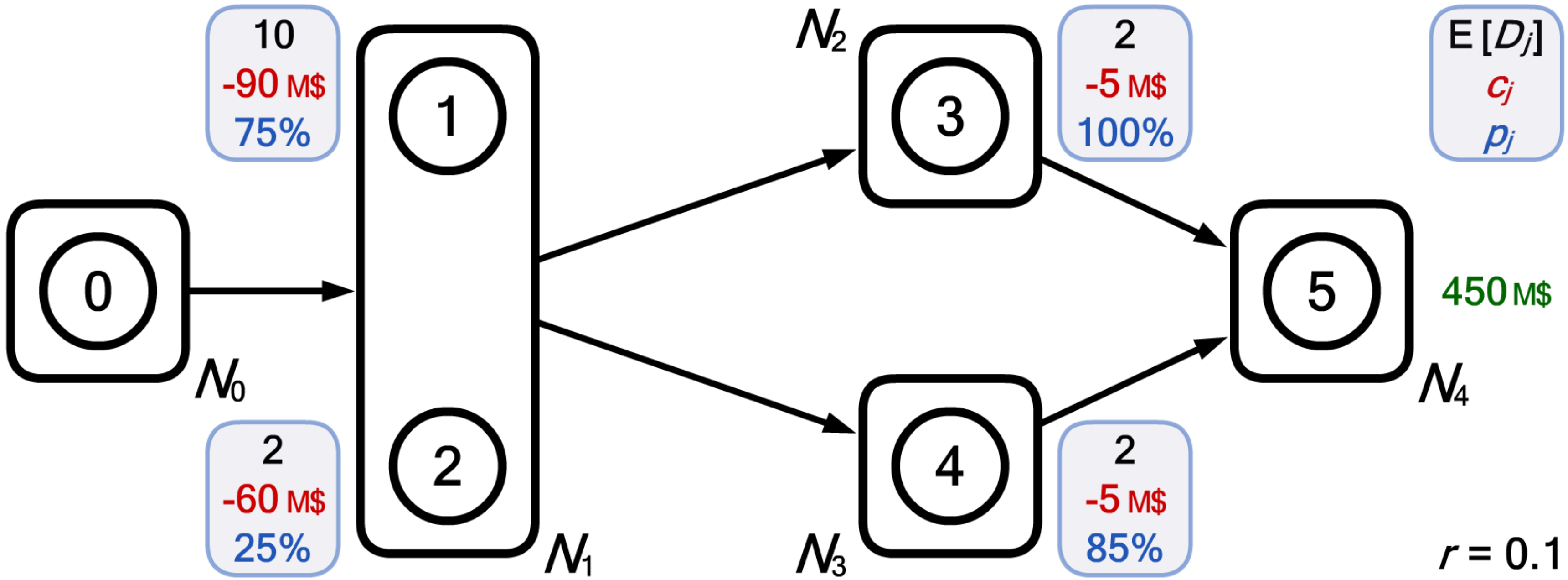
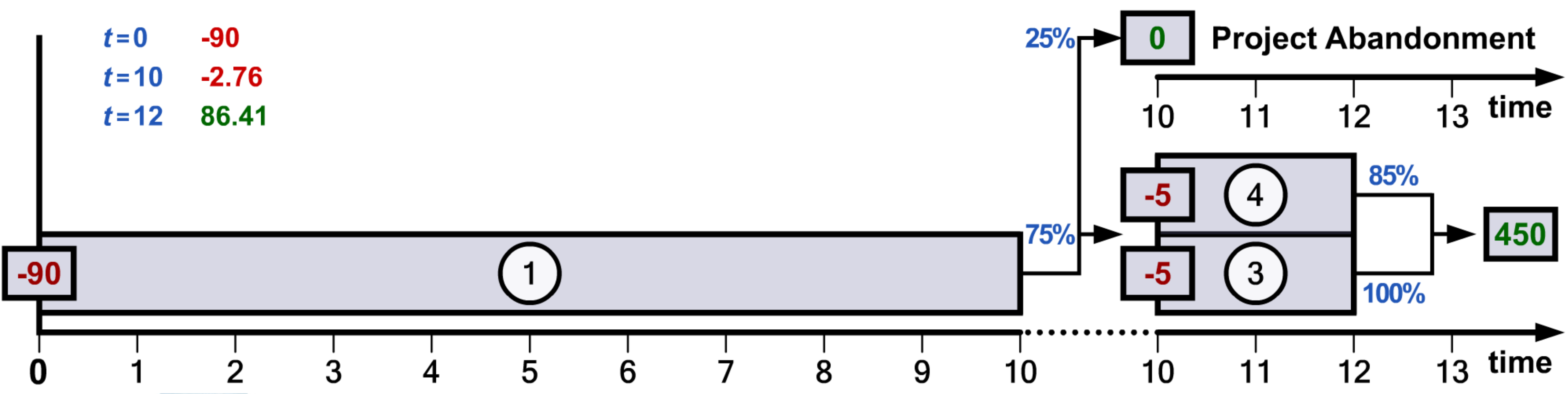
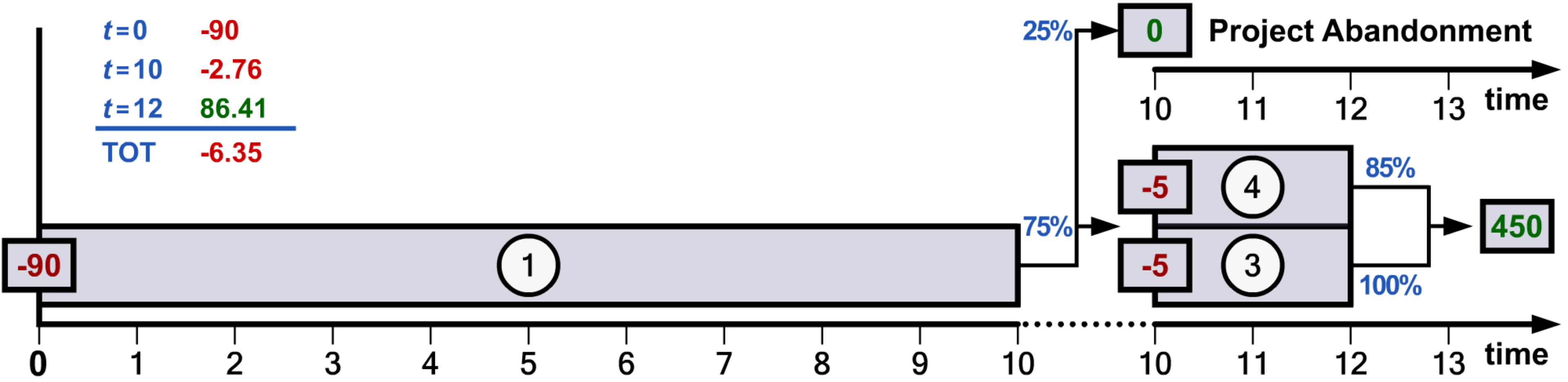




Deterministic durations

A solution is not a schedule but rather a scheduling policy (even with deterministic durations)

Policy Π_1 results in a NPV of -6.35M$ if activity durations are deterministic



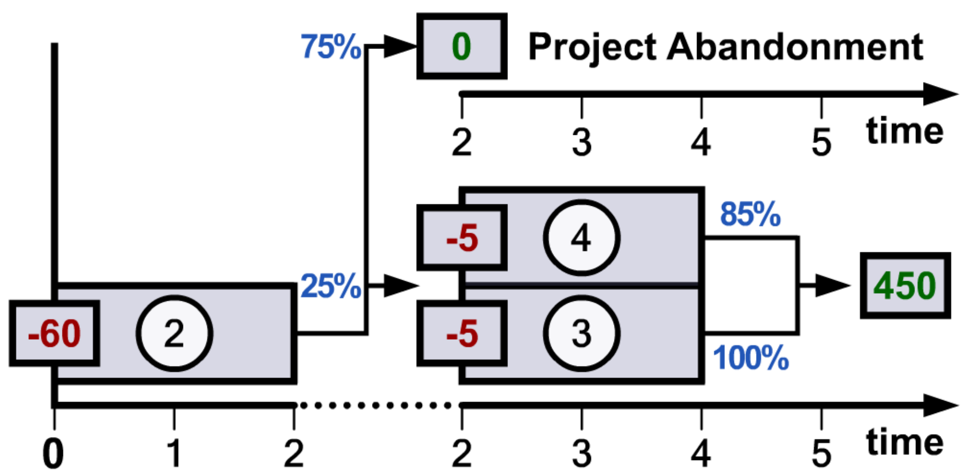
Deterministic durations

A solution is not a schedule but rather a scheduling policy (even with deterministic durations)

Policy Π_1 results in a NPV of -6.35M$ if activity durations are deterministic

Policy Π_2 is optimal for deterministic durations and yields a NPV of 2.05M$

$t=0$	-60
$t=2$	-2.04
$t=4$	64.10
TOT	2.05



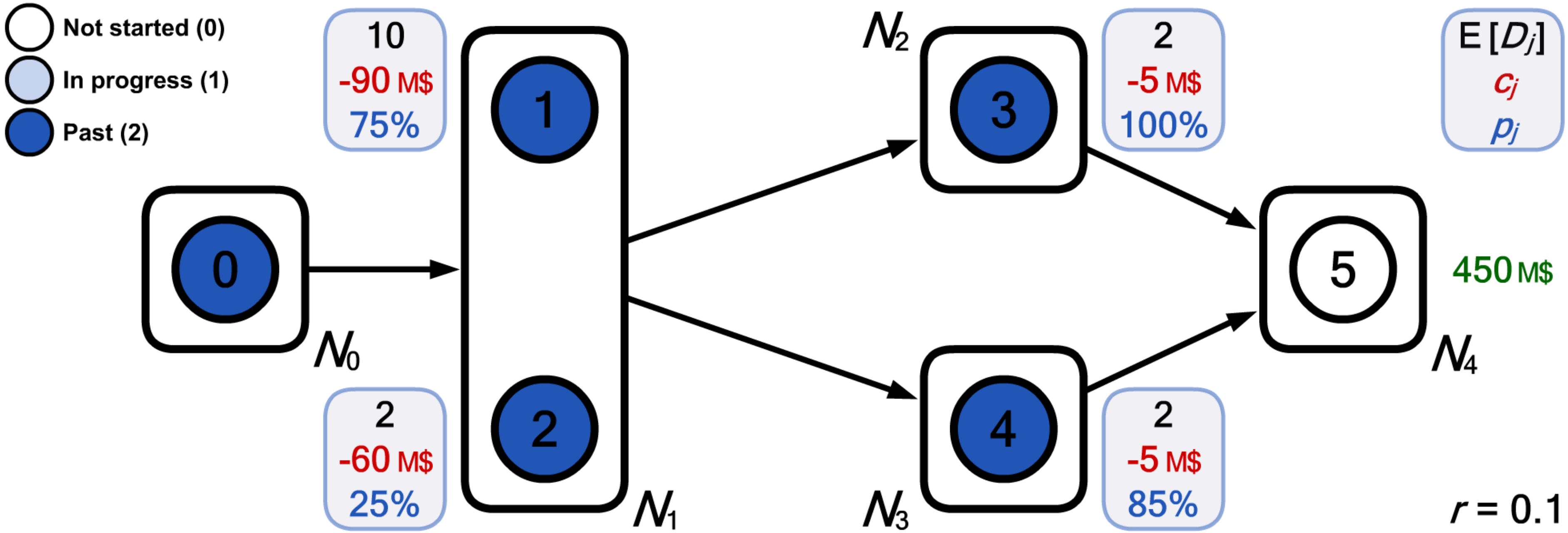
Stochastic durations: methodology

- Exponentially distributed durations => use of a Continuous-Time Markov Chain (CTMC) to model the statespace
- State of an activity *j* at time *t* can be:
  - Not started
  - In progress
  - Past (successfully finished, failed or considered redundant because another activity of its module has completed successfully)
- Size of statespace has upper bound 3^n. Most states do not satisfy precedence constraints => a strict definition of the statespace is required. This is studied in Creemers et al. (2010)*

⇒ Backward SDP-recursion

*Creemers S, Leus R, Lambrecht M (2010). Scheduling Markovian PERT networks to maximize the net present value. Operations Research Letters, vol. 38, no. 1, pp. 51 - 56.

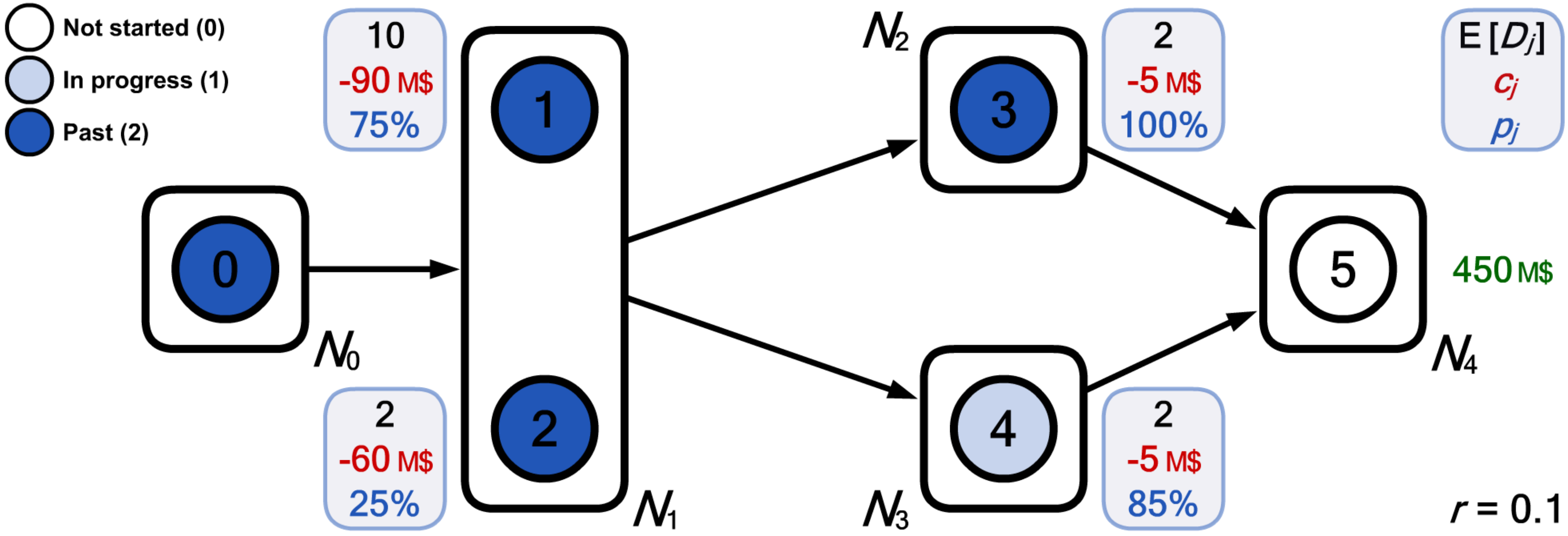
Stochastic durations



(2,2,2,2,2,0) [450M$]

Project value upon entry of the final state = project payoff

Stochastic durations



(2,2,2,2,2,0) [450M$]

→ (2,2,2,2,1,0) [318.75M$]

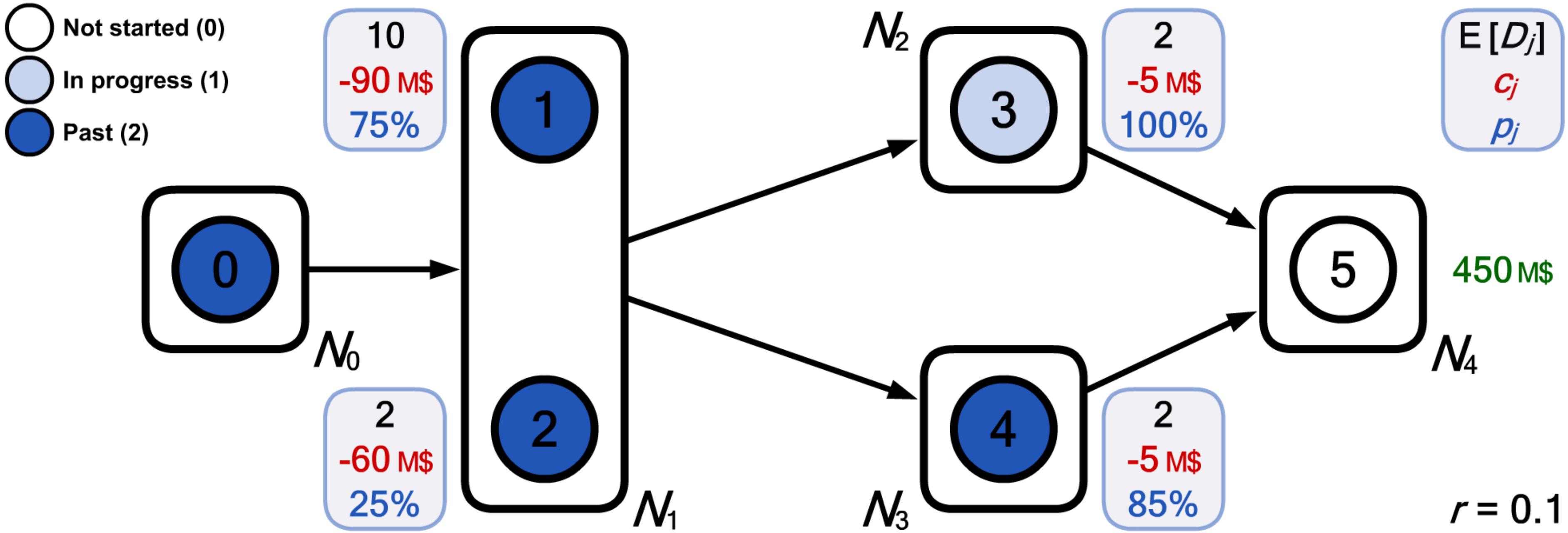
Discount factor: (1/E[D_4]) / (r+(1/E[D_4]))

D_4 = 2 => discount factor = 0.83

NPV upon state entry if success = 375

p_4 = 0.85 => NPV upon state entry = 318.75

Stochastic durations



(2,2,2,2,2,0) [450M$]
- (2,2,2,2,1,0) [318.75M$]
- (2,2,2,1,2,0) [375M$]

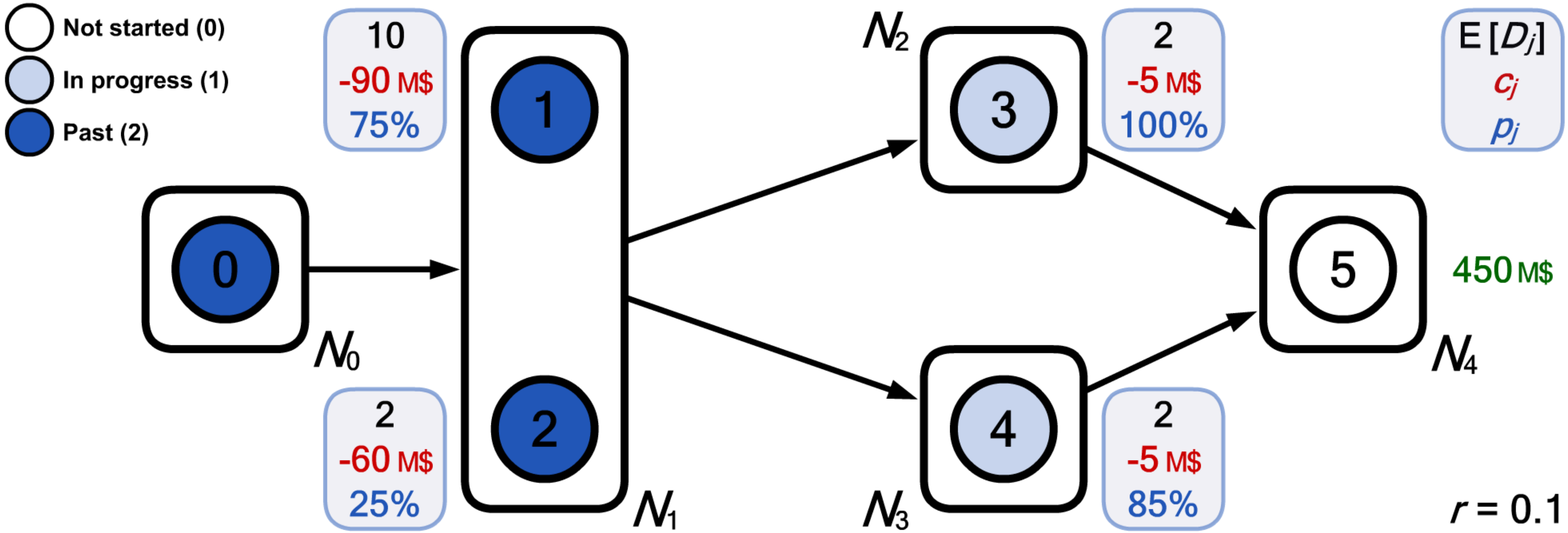
Discount factor: $(1/E[D_3]) / (r+(1/E[D_3]))$

D_3 = 2 => discount factor = 0.83

NPV upon state entry if success = 375

p_3 = 1.00 => NPV upon state entry = 375

Stochastic durations



(2,2,2,2,2,0) [450M$]

→ (2,2,2,2,1,0) [318.75M$]

→ (2,2,2,1,2,0) [375M$]

  → (2,2,2,1,1,0) [289.77M$]

Discount factor = 0.91

Probability of finishing activity j first : $(1/E[D_j]) / (\Sigma_k(1/E[D_k]))$

=> Probability 3 finishes first is 50% and p_3 = 100%

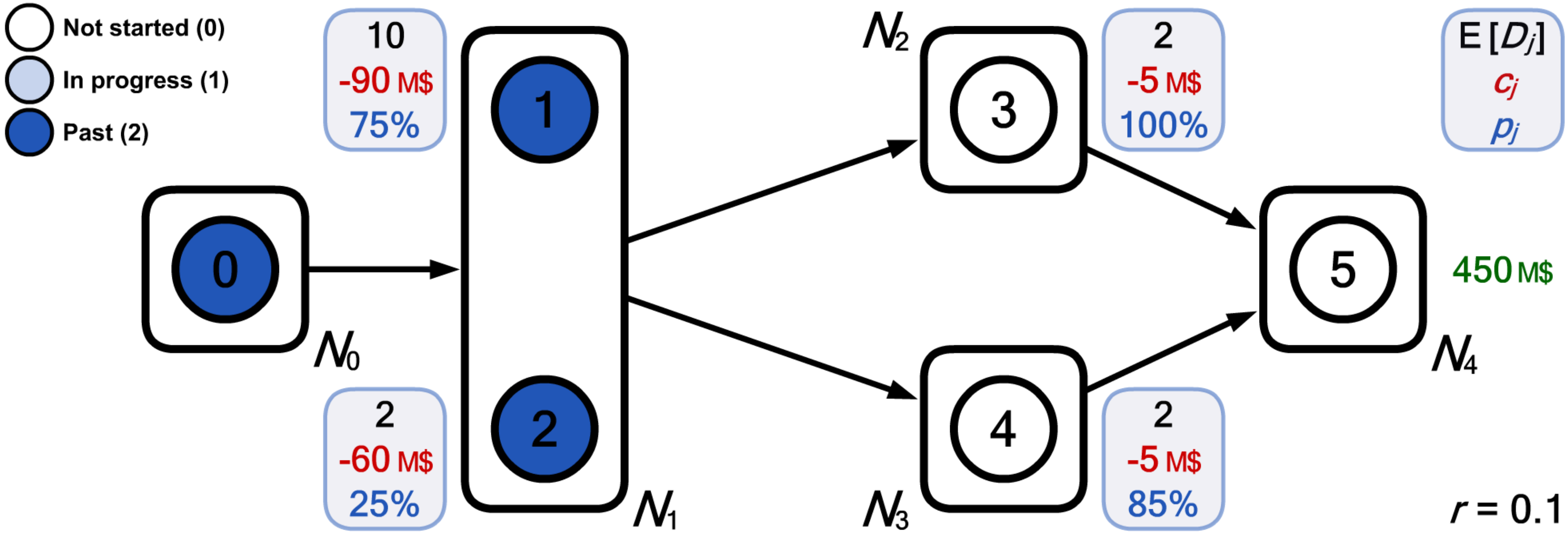
0.5 x 0.91 x 1.00 x 318.75 = 144.89

=> Probability 4 finishes first is 50% and p_4 = 85%

0.5 x 0.91 x 0.85 x 375 = 144.89

=> NPV upon state entry = 289.77

Stochastic durations



(2,2,2,2,2,0) [450M$]
→ (2,2,2,2,1,0) [318.75M$]
→ (2,2,2,1,2,0) [375M$]
→ (2,2,2,1,1,0) [289.77M$]
→ (2,2,2,0,0,0) [279.77M$]

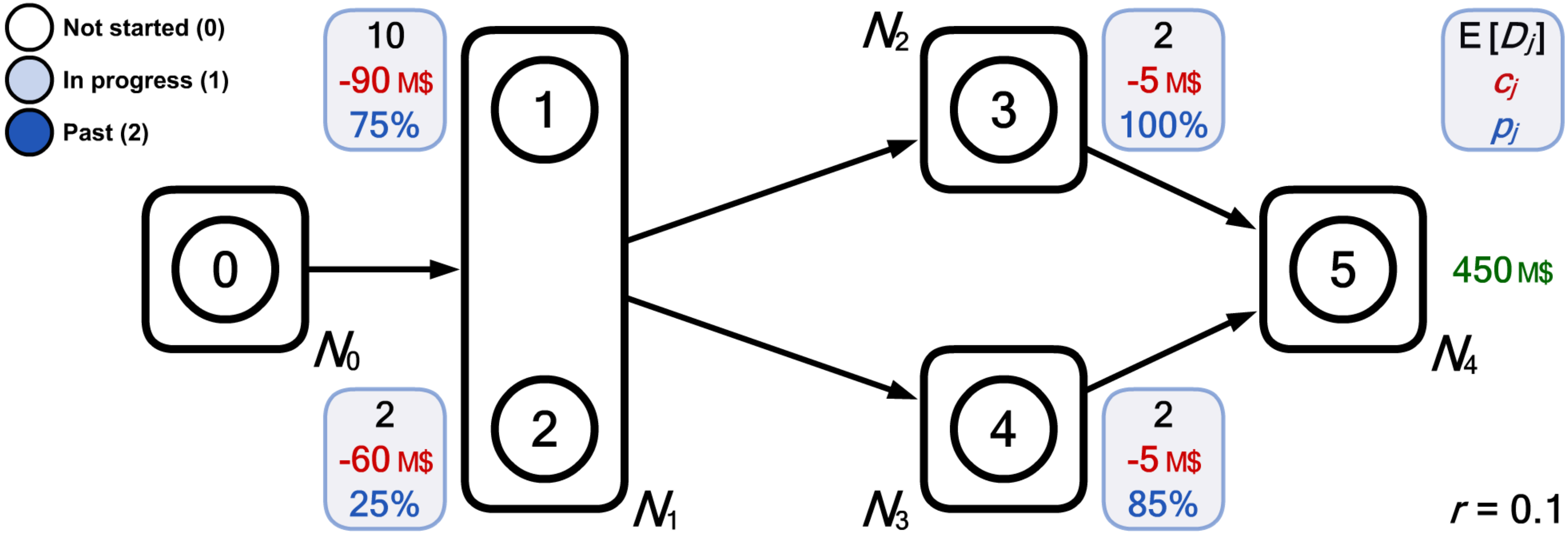
3 possible decisions (pick the optimal one):

- Start activity 3 => incur cost c_3 = -5M$
=> end up in (2,2,2,1,0,0)

- Start activity 4 => incur cost c_4 = -5M$
=> end up in (2,2,2,0,1,0)

- Start activity 3 & 4 => incur cost $c_3 + c_4$ = -10M$
=> end up in (2,2,2,1,1,0)[289.77M$]

Stochastic durations



(2,2,2,2,2,0) [450M$]
- → (2,2,2,2,1,0) [318.75M$]
- → (2,2,2,1,2,0) [375M$]
  - → (2,2,2,1,1,0) [289.77M$]
    - → (2,2,2,0,0,0) [279.77M$]
      - → (...)
        - → (0,0,0,0,0,0) [14.70M$]

Stochastic durations

A solution is not a schedule but rather a scheduling policy (even with deterministic durations)

Policy Π_1 results in a NPV of **-6.35M$** if activity durations are deterministic

Policy Π_2 is optimal for deterministic durations and yields a NPV of **2.05M$**

For stochastic durations, policy Π_1 is optimal with a NPV of **14.70M$**

- (2,2,2,2,2,0) [450M$]
  - (2,2,2,2,1,0) [318.75M$]
  - (2,2,2,1,2,0) [375M$]
    - (2,2,2,1,1,0) [289.77M$]
      - (2,2,2,0,0,0) [279.77M$]
        - (...)
          - (0,0,0,0,0,0) [14.70M$]

Results & future work

- Computational results:
  - 1260 randomly generated projects have been solved to optimality

n	10	20	30	60	90
CPU (sec)	0.00	0.03	1.95	84.04	4100.52

  - Main determinant of computation time = network density (for fixed *n*)
- Future work:
  - Using the model to generate insights
  - General activity durations using Phase-type distributions
  - Renewable resources

Questions?